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REVIEW

Review on chaotic dynamics of memristive neuron and neural network

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Abstract The study of dynamics on artificial neurons and neuronal networks is of great significance to understand brain functions and develop neuromorphic systems. Recently, memristive neuron and neural network models offer great potential in the investigation of neurodynamics. Many chaotic dynamics including chaos, transient chaos, hyperchaos, coexisting attractors, multistability, and extreme multistability have been researched based on the memristive neurons and neural networks. In this review, we firstly introduce the basic definition of chaotic dynamics and review several traditional artificial neuron and neural network models. Then we categorize memristive neuron and neural network models with different biological function mechanisms into five types: memristive autapse neuron, memristive synapse-coupled bi-neuron network, memristive synaptic weight neural network, neuron under electromagnetic radiation, and neural network under electromagnetic radiation. The modeling mechanisms of each type are explained and described in detail. Furthermore, the pioneer works and some recent important papers related to those types are introduced. Finally, some open problems in this field are presented to further explore future work.

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1 Introduction

Numerous physiological experiments show that electrical activities in biological neurons and nervous systems are closely associated with the brain's unique abilities including memory, thinking, and learning [1,2]. The mysteries of the brain have inspired many researchers to investigate the working mechanism of neural electrical activities. Various artificial neuron and neural network models have been developed around the 1980s to mimic the different electrical activities in biological neurons and neural systems, such as Hodgkin–Huxley (HH) neuron model [3], FitzHugh–Nagumo (FHN) neuron model [4,5], Morris–Lecar (ML) neuron model [6], Hindmarsh–Rose (HR) neuron model [7], Chay neuron model [8], Hopfield neural network (HNN) [9], Cellular neural network (CNN) [10], and so on [11]. From then on, some neural electrical activities including periodic spiking [12], periodic bursting [13], and mode transition [14] have been investigated based on these original neuron and neural network models as well as their extended versions like the delay time models [15], noise models [16], and electrical stimulus models [17,18]. Though the simple electrical activities were reproduced from these traditional neuron and neural network models, more complex dynamics like chaotic behaviors were not discovered then.

A renewed research interest in neurodynamics was generated when a physical memristor device was found in 2008 [19]. Over the past 10 years, memristor with biomimetic characteristics has made extraordinary development, which has a great impact on artificial neuron and neural network models. Due to the intrinsic properties such as programmability, nonvolatility, nanoscale, and low-power, the memristor gives us unprecedented possibilities in imitating biological functions. For example, Jo et al. [20] proposed the first nanoscale memristive synapse that exhibits spike-timing-dependent plasticity, and Lv et al. [21] firstly described the magnetic flow effect in neurons by using a voltage-controlled memristor. Memristor is added with traditional neuron and neural network models in giving a new concept known as memristive neuron and neural network models. The involvement of memristor in traditional neuron and neural network models can trigger complex chaotic dynamics, which is now attracted by different researchers from all most every part of the world. Various memristive neuron and neural network models have been developed based on different modeling mechanisms. In 2010, Pershin and Ventra [22] experimentally demonstrated a simple memristive neural network with three neurons connected by two memristor synapses. Li et al. [23] in 2014 proposed a small memristive synaptic weight HNN by using the memductance of the memristor to replace the synaptic weight. Lv and Ma [24] in 2016 given a memristive HR neuron model via utilizing a magnetic-controlled memristor to describe the effect of electromagnetic radiation. By using a memristor as a neural synapse between two neurons, Xu et al. [25] constructed a memristive synapse-coupled FHN bi-neuron network in 2017. Since the induction current can be described by memristor, Bao et al. [26] in 2018 created a memristive HR neuron model by considering the electromagnetic induction effect caused by the membrane potential of the neuron. Based on the effect of a magnetic field in a single neuron, Hu et al. [27] proposed a memristive HNN under electromagnetic radiation in the same year. Moreover, in 2020, Lin et al. [28] established a locally active memristive neuron model by using a locally active memristor as a feedback autapse.

It has been shown that the memristive neuron and neural network models can generate complex chaotic behaviors. For example, for the first time, chaotic spiking and bursting firings are observed in memristive HR neurons [29,30]. The phenomena of chaos and synchronization is obtained in memristive bi-neuron networks [31–34]. And various complex chaotic phenomena including hyperchaos [35,36], hidden attractors [37–39], coexisting attractors [40–44], and multi-scroll attractors [45,46] are discovered in various memristive neurons and neural networks. Also, the complicated dynamics of multistability [28,47–50] and extreme multistability [35,51,52] have been reported. Undoubtedly, the realization of these complex neurodynamics will be helpful in clinical aspects [53,54]. Besides, the memristive neuron and neural network models owning complex dynamics can be better applied in artificial intelligence fields, especially secure communication [55]. For instance, Wang et al. [56] proposed an image encryption and decryption scheme based on a memristive Hopfield neural network with chaotic behavior. And Guo et al. [57] solved static and dynamic image associative memory in a multi-later memristive recurrent neural network. In short, memristive neurons and neural networks have received extensive attention due to the combination of abundant dynamics and wide applications.

In this paper, we review the memristive neuron and neural network models from the perspective of chaotic dynamics. In Sect. 2, we have given the basic concept of chaotic dynamics. Some traditional artificial neuron and neural network models are summarized in Sect. 3. In Sect. 4, different memristive neurons and neural networks are explained and discussed in detail. Some open problems and future work are presented in Sect. 5. Finally, in Sect. 6, conclusions are drawn.

2 Basic concept of chaotic dynamics

Neurodynamics is an interdisciplinary subject of neuroscience and dynamical system theory [58]. Its purpose is to study the dynamical characteristics of biological nervous systems and their evolution over time, especially the firing behavior, chaotic property, and the phenomena of synchronization and bifurcation by applying dynamical system theory, especially the thought and method of nonlinear dynamics. From the perspective of chaotic dynamics, the dynamical characteristics include chaos, transient chaos, and hyperchaos. Chaos is a special dynamical behavior [59,60], which exists widely in all kinds of natural nonlinear systems, particularly biological neural systems [61]. According to the theory of Lyapunov, chaos has at least one positive Lyapunov exponent. And transient chaos is a dynamical behavior that the existence of chaos is on finite time [62]. Furthermore, hyperchaos is defined as chaos with two or more positive Lyapunov exponents [63], which is more complicated than chaos. Generally speaking, the dynamical trajectory of chaos is called attractors. An attractor is called a hidden attractor if its basin of attraction does not intersect with any open neighborhood of the system equilibria, or otherwise, it is called a self-excited attractor [64,65]. Generally, multi-scroll/wing attractors are more complex compared with single-scroll/wing attractors [66,67]. From the perspective of stability, the chaotic dynamics contain coexisting attractors [68,69], multistability [70,71], and extreme multistability [72–74]. The phenomenon of coexisting attractors is an intricate dynamical phenomenon that contains two types of different chaotic behaviors under two different initial states. The coexistence of three or more dynamical states under different initial states is called as multistability. Multistability means that a rich diversity of stable states exists in a nonlinear system, which reflects the characteristics of complex systems. And the phenomenon of the coexistence of infinitely many attractors is called extreme multistability. Additionally, from the perspective of the time domain, chaotic dynamics can be divided into chaotic spiking, chaotic bursting, and chaos firing [28]. Generally, chaotic spiking includes different periods or amplitudes spikes. Chaotic bursting includes different numbers or amplitudes of bursts. And chaos firing is a completely chaotic ruleless time sequence. It should be stressed that all chaotic spiking, chaotic bursting, and chaos firing are chaotic behaviors with at least one positive Lyapunov exponent.

3 Traditional neuron and neural network models

Artificial neuron and neural network models have made a great contribution to the development of neurodynamics. In this section, various classical artificial neuron models and neural networks are summarized.

3.1 HH neuron model

In 1952, Hodgkin and Huxley (HH) [3] first constructed a nonlinear dynamical system as a mathematical model of single-neuron based on their electrophysiological experiments with squid giant atoms. The HH neuron model describes the spiking behavior and refractory properties of real neurons and serves as a paradigm for spiking neurons based on the nonlinear conductance of ion channels. The model is given by four nonlinear coupled equations, one for the membrane potential V, and three for gating variables, m, n, and h: [75]

$$\begin{cases} C_m \mathrm{d}V/\mathrm{d}t = I_{\mathrm{ion}} + I_{\mathrm{syn}} + I_{\mathrm{ext}} \\ \mathrm{d}m/\mathrm{d}t = \frac{m_\infty(V)-m}{\tau_m(V)} \\ \mathrm{d}h/\mathrm{d}t = \frac{h_\infty(V)-h}{\tau_h(V)} \\ \mathrm{d}n/\mathrm{d}t = \frac{n_\infty(V)-n}{\tau_n(V)} \end{cases} \tag{1}$$

where

$$\begin{aligned} I_{\mathrm{ion}} &= -g_{\mathrm{Na}} m^3 h(V - V_{\mathrm{Na}}) - g_{\mathrm{K}} n^4 (V - V_{\mathrm{K}}) \\ &\quad - g_L (V - V_L) \end{aligned} \tag{2}$$

where I_{ion}, I_{syn} and I_{ext} represent ionic current, synaptic current, and external stimulus, respectively. The ionic current I_{ion} is related to the gating variables of m, n, h and describes the ionic transport through the membrane. The constants g_{Na}, g_{K}, and g_L are the maximal conductances for sodium ion (Na^+), potassium ion (K^+) and leakage channels, and V_{Na}, V_{K}, V_L are the corresponding reversal potentials. $m_\infty, h_\infty, n_\infty$ and τ_m, τ_h, τ_n represent the saturation values and the relaxation times of the gating variables.

3.2 FHN neuron model

In 1962, the two-dimensional FitzHugh–Nagumo (FHN) model which is simplified from the four-dimensional HH model was introduced to describe neuronal excitability and spiking firing [4,5]. The FHN model reflects the main characteristics of neuron firing activity and can be described by [76]

$$\begin{cases} \mathrm{d}x/\mathrm{d}t = \frac{1}{a}\left(x - \frac{x^3}{3} - y + I_{\mathrm{ext}}\right) \\ \mathrm{d}y/\mathrm{d}t = ax - by + c \end{cases} \tag{3}$$

where x is the membrane potential (fast variable), y is the ion current (slow variable), and I_{ext} is the external stimulus. The constant a, b, c are model parameters.

3.3 ML neuron model

In 1981, Morris and Lecar (ML) [6] proposed a simplified HH neuron model called ML model. The ML neuron model is a biological neuron model developed to reproduce the variety of oscillatory behavior in relation to calcium ion (Ca^{2+}) and K^+ conductance in the giant barnacle fibers. This model is a two-dimensional system of nonlinear differential equations: [77]

$$\begin{cases} C\mathrm{d}V/\mathrm{d}t = -g_{\mathrm{Ca}}M_\infty(V)(V-V_{\mathrm{Ca}}) \\ \quad -g_{\mathrm{K}}W(V-V_{\mathrm{K}}) - g_L(V-V_L) + I_{\mathrm{ext}} \\ \mathrm{d}W/\mathrm{d}t = \tau_W(W_\infty(V)-W) \end{cases} \tag{4}$$

where

$$\begin{cases} M_\infty(V) = 0.5 + 0.5\tanh\left(\frac{V-V_1}{V_2}\right) \\ W_\infty(V) = 0.5 + 0.5\tanh\left(\frac{V-V_3}{V_4}\right) \\ \tau_W(V) = \frac{1}{\tau_W}\cosh\left(\frac{V-V_3}{2V_4}\right) \end{cases} \tag{5}$$

where V and W represent the variables for the membrane potential and gate channel, respectively. C is the capacitance of the membrane, and g_{Ca}, g_{K}, and g_L denote maximum conductance of Ca^{2+}, maximum conductance of K^+, and maximum conductance of leakage current, respectively. I_{ext} is an external stimulus. V_{Ca}, V_{K} and V_L are steady-state potentials for Ca^{2+}, K^+ and leak ion channels, respectively. $M_\infty(V)$ and $W_\infty(V)$ define the stable values of opening probability for calcium and potassium, where V_1, V_2, V_3, and V_4 are parameters of steady states, and τ_w is the system parameter.

3.4 HR neuron model

In 1984, Hindmarsh and Rose (HR) [7] developed a powerful HR model which can not only facilitate the calculation but also can generate most of the firing behaviors exhibited by real biological neurons, such as quiescence, spiking firing, and bursting firing. The HR model includes a 2D model and a 3D model. The 2D HR neuron model is regarded by many scholars as to the idealistic one in the study of actual neuron firing. Its mathematical expression is [78]

$$\begin{cases} \mathrm{d}x/\mathrm{d}t = y - ax^3 + bx^2 + I_{\mathrm{ext}} \\ \mathrm{d}y/\mathrm{d}t = c - dx^2 - y \end{cases} \tag{6}$$

where x and y denote membrane potential and recovery variables of the neuron, a, b, c, and d are model parameters, and I_{ext} is the external stimulus. The 3D HR neuron model is described by the following dynamical system: [79]

$$\begin{cases} \mathrm{d}x/\mathrm{d}t = y - ax^3 + bx^2 - z + I_{\mathrm{ext}} \\ \mathrm{d}y/\mathrm{d}t = c - dx^2 - y \\ \mathrm{d}z/\mathrm{d}t = r(s(x+\varepsilon) - z) \end{cases} \tag{7}$$

where the state variable x represents the membrane potential, y describes the exchange of ions across the neuron membrane through fast ionic channels, and z is a slowly changing adaptation current. I_{ext} mimics the external current for biological neurons, and r is a small parameter that controls the speed of variation of the slow variable z, ε sets the resting potential of the system. And a, b, c, d, s, r are system parameters.

3.5 Chay neuron model

In 1985, to reproduce the firing behaviors of β-cell, Chay [8] developed a three-dimensional neuron model that can simulate bursting and chaos firing. The Chay model is described using the following three differential equations: [80]

$$\begin{cases} \mathrm{d}V/\mathrm{d}t = -I_{\mathrm{ion}} - I_{\mathrm{kv}} - I_{\mathrm{kc}} - I_L + I_{\mathrm{ext}} \\ \mathrm{d}n/\mathrm{d}t = (n_\infty - n)/\tau_n \\ \mathrm{d}C/\mathrm{d}t = \rho(m_\infty^3 h_\infty(V_c - V) - k_c C) \end{cases} \tag{8}$$

where

$$\begin{cases} I_{\mathrm{ion}} = g_{\mathrm{ion}} m_\infty^3 h_\infty^3 (V - V_{ion}) \\ I_{kv} = g_{kv} n^4 (V - V_{\mathrm{k}}) \\ I_{kc} = g_{kc}(C/(1+C))(V - V_{\mathrm{k}}) \\ I_L = g_L(V - V_L) \end{cases} \tag{9}$$

where V, n, and C are the membrane potential, the probability of opening voltage-dependent K^+ channels, and the intracellular Ca^{2+} concentration, respectively. I_{ion}, I_{kv}, and I_{kc} are the inward mixed Na^+–Ca^{2+} ionic current, the outward voltage-dependent K^+ ionic current, and the outward calcium-dependent K^+ ionic current, respectively. And I_L and I_{ext} are the leakage current and external stimulus, respectively. V_{ion}, V_{K}, and V_L are reversal potentials for mixed Na^+–Ca^{2+}, K^+, and leakage ions, respectively. g_{ion}, g_{kv}, g_{kc}, and g_L represent the maximal conductances, where the subscripts refer to the voltage-dependent mixed ionic channel, the voltage-dependent K^+ channel, the Ca^{2+}-dependent K^+ channel, and leakage channels, respectively. m_∞ and h_∞ in Eq. (9) are the probabilities of activation and inactivation of the mixed channel, respectively. n_∞ is the steady-state value of n.

According to Sects. 3.1–3.5, all single neuron models can be simplified as a common neuron model, namely

$$\begin{cases} \mathrm{d}V_{\mathrm{mem}}/\mathrm{d}t = f(V_{\mathrm{mem}}, V_y, \ldots, I_{\mathrm{ext}}) \\ \mathrm{d}V_y/\mathrm{d}t = f(V_{\mathrm{mem}}, V_y, \ldots) \\ \vdots \end{cases} \tag{10}$$

where V_{mem} represents the membrane potential of the neuron, various other V_y represent some state variables induced by various ion channels. I_{ext} is the external stimulus.

3.6 Hopfield neural network

In 1984, Hopfield [9] proposed an ideal neural network, namely Hopfield neural network (HNN). Due to its strong nonlinear and flexible algebraic expression, the HNN is particularly suitable for simulating various complex dynamical behaviors in the brain, especially chaotic behavior. HNN can be described by a set of nonlinear ordinary differential equations corresponded to n-neurons. The mathematical model of HNN can be given by [81]

$$C_i \mathrm{d}x_i/\mathrm{d}t = -x_i/R_i + \sum_{j=1}^{n} w_{ij} v_j + I_{i\text{ext}} \tag{11}$$

where C_i, R_i, x_i are membrane capacitance, membrane resistance, and membrane potential of ith neuron, respectively. w_{ij} is the synaptic weight between the ith and jth neurons. v_j=tanh(x_j) is the neuron activation function, and I_{iext} is the external stimulus.

3.7 Cellular neural network

In 1988, Chua and Yang [10] presented a Cellular neural network (CNN) based on cellular automata and Hopfield neural networks. The basic unit circuit of CNN is called a cell. The system equation for an $M \times N$ CNN is [82]

$$\begin{cases} C\mathrm{d}x_{ij}/\mathrm{d}t = -x_{ij}/R + \sum\limits_{C(k,l)\in N_r(i,j)} A(i,j;k,l)y_{kl} \\ \quad + \sum\limits_{C(k,l)\in N_r(i,j)} B(i,j;k,l)u_{kl} + I_{ij\text{ext}} \\ y_{ij} = 0.5(|x_{ij}+1| - |x_{ij}-1|) \end{cases} \tag{12}$$

where C and R are linear capacitor and linear resistor, respectively. x_{ij}, y_{ij}, u_{ij} are internal state, output, input of neuron (i, j), respectively. A $(i, j; k, l)$ and B $(i, j; k, l)$ are output feedback parameter and input control parameter, respectively. I_{ijext} is external input current. C (k, l) represents neuron (k, l), and Nr (I, j) represents the r-neighborhood of neuron (I, j), namely,$\{(k,l)|\max\{|k-i|, |l-j|\} \le r, 1 \le k \le M, 1 \le l \le N\}$. Additionally, $|xij(\theta)| \le 1$ and $|uij| \le 1$.

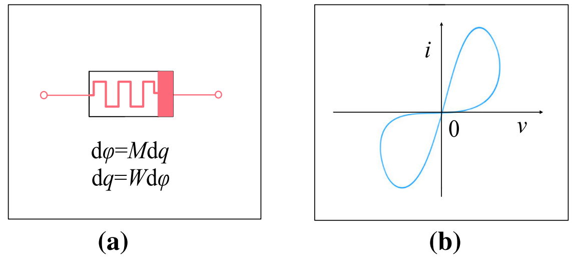

Fig. 1 Circuit symbol and pinched hysteresis loop of the memristor. **a** Circuit symbol. **b** Pinched hysteresis loop

4 Memristive neuron and neural network models

Memristor was postulated first by Chua in 1971 [83] as the fourth basic electrical element that links flux (φ) and charge (q). Later the concept is generalized to any two-terminal device exhibiting a pinched hysteresis loop which always passes through the origin in the voltage–current plane when driven by any periodic voltage or current signal that results in a periodic response of the same frequency [84–86]. The symbol of the memristor and its unique pinched hysteresis loop are shown in Fig. 1a and b, respectively, where M and W represent memristance and memductance, respectively. Although the concept of memristor was introduced in the early time, an actual physical memristor was discovered until 2008 [19]. After that, various different devices have been identified as memristors, and corresponding memristor models that can mimic the approximately the measured pinched hysteresis loops have been developed and applied [87,88].

According to the theory of the memristor, a common ideal memristor model can be written by

$$\begin{cases} i = W(\varphi)v \\ \mathrm{d}\varphi/\mathrm{d}t = v \end{cases} \tag{13}$$

where v, i are the input voltage and output current, respectively. W is a continuous function of φ, called the memductance, and φ is the flux. And a generic memristor is defined by

$$\begin{cases} i = W(x)v \\ \mathrm{d}x/\mathrm{d}t = f(x, v) \end{cases} \tag{14}$$

where W is a continuous function of x, called the memductance, x is the state variable, and $f(x, v)$ is a Lipschitz function.

As we all know, a memristor is a two-terminal electronic device whose memductance can be precisely modulated by charge or flux through it. Furthermore,

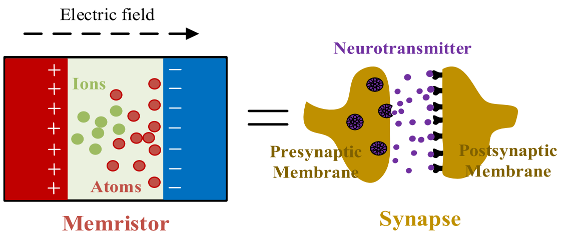

Fig. 2 Schematic illustration of the concept of using memristor as neural synapse

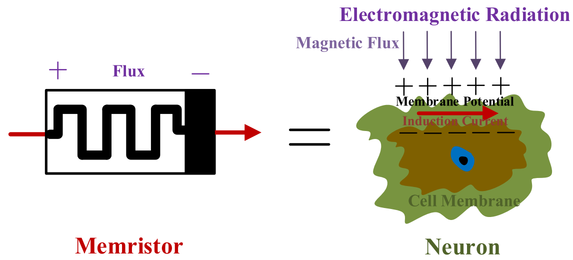

Fig. 3 Schematic illustration of the concept of using memristor to describe the coupling between magnetic flux and membrane potential

the memristor enjoys many biomimetic features such as nanoscale, nonlinearity, programmability, and memorability [89]. Therefore, the memristor has two important biomimetic functions. On the one hand, the memristor can be used to imitate synaptic functions such as plasticity. Numerous research results show that the mechanism of moving nanoscale particles in memristors is very similar to the behavior of moving neurotransmitters in synapses [90]. Thus, a memristor can be considered as an artificial neural synapse, as shown in Fig. 2. Also, many experimental results demonstrated that nanoscale memristor devices can support synaptic plasticity [91,92]. As a result, using memristors as synapses in neuron and neural network models can establish more realistic artificial neuron and neural network models.

On the other hand, the memristor can be used to describe the effect of electromagnetic radiation on neuronal electrical activity. When a neuron is exposed to external electromagnetic radiation, the motion of ions with charge can be controlled by the electromagnetic field, and an electromagnetic induction current can be generated [21,24]. Hence, the effect of electromagnetic radiation can be considered as magnetic flux across the membrane of the neuron, and a memristor can be used to describe the coupling between magnetic flux and membrane potential, as shown in Fig. 3. With electromagnetic radiation being considered, the traditional neuron model can present more abundant chaotic dynamics.

Therefore, memristors can be employed to mimic biological neural synapses or to describe electromagnetic induction effects caused by external electromagnetic radiation. With the advent of memristors, many types of traditional neuron and neural network models have been improved by memristors. Next, we introduce various memristive neuron and neural network models and discuss how these models simulate real biological neurons and nervous systems. Especially, we review the recent papers related to the chaotic dynamics of various memristive neurons and memristive neural networks.

4.1 Memristive autapse neuron model

As it is well known, a neuron is made up of the nucleus encoding information, the dendrite collecting electrical signals, and the axon propagating electrical signals. Synapse is an important bridge for connecting the axon and the dendrite of different neurons, which plays a key role in receiving and transferring electrical signals between neurons. Based on some physical and biological experiments, electrical activities can be modulated by the synapse current. Autapse can connect the axon and the dendrite of the same neuron by a close loop, which is a type of special synapse [93]. As reported in [94], autapse can regulate neuronal activity by a negative feedback autapse current. Therefore, it is significant and necessary to consider autapse as a part of a neuronal system. A great number of researches show that biological neural synapse owning memory characteristic is considered as a kind of memristor device [95]. Therefore, the memristor can efficiently mimic the neural synapses. Under this strategy, introducing an autapse current generated by the memristor-based autapse into the traditional single neuron model, a memristive autapse neuron model can be established [28]. The reduced diagram of the memristive autapse neuron is given in Fig. 4. As shown in Fig. 4, the memristive autapse current can be computed by an induction current caused by memductance and membrane potential, namely

$$I_{\text{aut}} = \rho V_{\text{mem}} W(x) \tag{15}$$

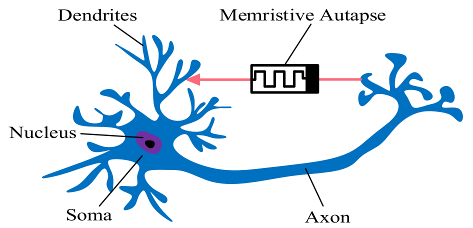

Fig. 4 Concept diagram of the memristive autapse neuron model

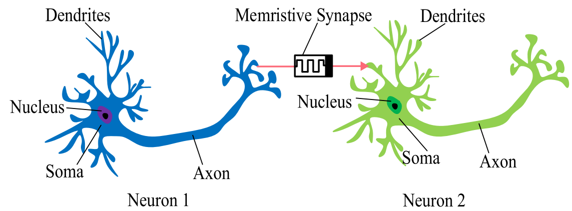

Fig. 5 Concept diagram of the memristive autapse neuron model

where V_{mem} is the membrane potential of the neuron, $W(x)$ is memductance of the memristor, and ρ indicates the feedback gain of the autapse current. When the autapse current I_{aut} in Equation (15) is considered in the traditional neuron model in Equation (10), the memristive autapse neuron model can be constructed by

$$\begin{cases} \mathrm{d}V_{mem}/\mathrm{d}t = f(V_{mem}, V_y, \ldots, I_{ext}) - I_{aut} \\ \mathrm{d}V_y/\mathrm{d}t = f(V_{mem}, V_y, \ldots) \\ \vdots \\ \mathrm{d}x/\mathrm{d}t = f(x, V_{mem}) \end{cases} \tag{16}$$

where V_{mem} is the membrane potential of the neuron, I_{aut} is often regarded as a negative feedback current, $\mathrm{d}x/\mathrm{d}t$ is the state function of the memristor, and x is the internal state of the memristor in Equation (14). Base on this type of memristive neuron model, many complex chaotic dynamics can be investigated [96,97]. For example, firing multistability including chaotic spiking and chaotic bursting is observed in the locally active memristive HR neuron [28]. Multiple firing modes can be generated in the memristive FHN neuron [98]. Moreover, the autapse current is also regarded as a kind of self-induction current caused by the membrane potential of the neuron. Hidden bursting firing [97] and coexisting behaviors [99] are found in the memristive HR neurons.

4.2 Memristive synapse-coupled bi-neuron network

Synapse plays an important role in the signal exchange and information encoding between neurons. Electric and chemical synapses are often used to investigate the synchronization and chaos in the electrical activities of neurons. Since memristor can mimic biological synapse, it can use to connect two neurons and construct a memristive synapse-coupled bi-neuron network. The simplified schematic diagram of the memristive-synapse coupled bi-neuron network is given in Fig. 5. As shown in Fig. 5, the signal exchange can be induced when two neurons are connected via memristor, namely, a synapse current can be generated on the memristive synapse. According to the nonlinear property of the memristor, the memristive synapse current can be described by

$$I_{syn} = \rho\,(V_{mem1} - V_{mem2})\,W(x) \tag{17}$$

where V_{mem1} and V_{mem2} denote two membrane potentials of neuron 1 and neuron 2. $W(x)$ and ρ are memductance of the memristor and the coupling strength between the memristor and neurons. According to the dynamical equation of the traditional neuron models, the memristive synapse-coupled bi-neuron network can be described by

$$\begin{cases} \mathrm{d}V_{mem1}/\mathrm{d}t = f(V_{mem1}, V_{y1}, \ldots, I_{ext1}) - I_{syn} \\ \mathrm{d}V_{y1}/\mathrm{d}t = f(V_{mem1}, V_{y1}, \ldots) \\ \vdots \\ \mathrm{d}V_{mem2}/\mathrm{d}t = f(V_{mem2}, V_{y2}, \ldots, I_{ext2}) + I_{syn} \\ \mathrm{d}V_{y2}/\mathrm{d}t = f(V_{mem2}, V_{y2}, \ldots) \\ \vdots \\ \mathrm{d}x/\mathrm{d}t = f(x, (V_{mem1} - V_{mem2})) \end{cases} \tag{18}$$

Under this mechanism, some synapse-coupled bi-neuron networks and their neurodynamics can be further analyzed. For instance, the phenomena of chaos [31,34] and synchronization [25,100–104] are detected in various memristive synapse-coupled bi-neuron networks. Coexisting multiple firing patterns are observed in the memristive synapse-coupled HR bi-neuron network [43,44,47].

4.3 Memristive synaptic weight neural network

By applying voltage or current to the memristor, its memductance can be altered like the biological synapse. And from the electronic point of view, both

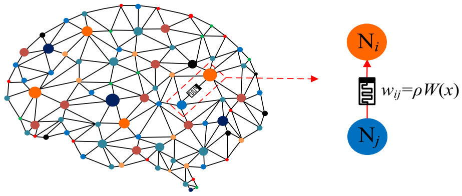

Fig. 6 Concept diagram of the memristive synaptic weight neural network

the synaptic weight of the synapse and memductance of the memristor are in the units of Siemens [20]. Therefore, the memductance is very suitable to realize the synaptic weight. Under this mechanism, a memristive synaptic weight neural network can be modeled based on the traditional neural networks. Figure 6 shows the conceptual structure of the memristive synaptic weight neural network, where the memristor is used to simulate the synapse between the ith neuron (N$_i$) and the jth neuron (N$_j$) in a neural network. As a result, the synapse weight w_{ij} of the synapse can be described by using $\rho W(x)$, where $W(x)$ and ρ represent the memductance and the coupling strength of the memristor, respectively.

According to the above analysis, assuming that the synapse weight w_{ij} in the Hopfield neural network is replaced by the memductance $\rho W(x)$, a memristive Hopfield neural network can be modeled by

$$\begin{cases} C_i \mathrm{d}x_i/\mathrm{d}t = -x_i/R_i + \sum\limits_{k=1,k\neq j}^{n} w_{ik}v_k + w_{ij}v_j + I_{iext} \\ \mathrm{d}x/\mathrm{d}t = f(x, v_j) \end{cases} \tag{19}$$

where $i, j \subset [1, n]$. In recent years, many scholars have devoted great enthusiasm to the memristive Hopfield neural network and its chaotic dynamics. For the first time, hyperchaotic behavior and hidden attractors are, respectively, revealed in two small memristive HNNs with a quadratic memristive synaptic weight [23,37]. Coexisting two asymmetric attractors are observed in a memristive HNN with a hyperbolic-type memristive synaptic weight [41]. The phenomena of coexisting multiple attractors and remerging Feigenbaum trees are discovered in a four neuron-based memristive HNN with a nonlinear memristive synaptic weight [49]. And coexistence of infinite attractors can be obtained from a four neuron-based memristive HNN with a multi-stable memristive synaptic weight [51]. Very recently, in [46], the initial offset boosting coexisting multi-double-scroll attractors are revealed in a memristive HNN with a non-ideal flux-controlled memristor synapse. Also, other memristive HNNs are also studied based on this model [105–107].

4.4 Neuron under electromagnetic radiation

With the development of modern industry, the wide utilization of electric equipment makes the biological nervous system be exposed to an environment full of electromagnetic radiation, which has a great influence on the dynamics of a single neuron and neuronal networks [2]. Indeed, the motion of the charged particle can be controlled by an electromagnetic field and the spatial distribution of charged particles becomes complex when these charged particles are exposed to the external electromagnetic field. According to the physical law of electromagnetic induction, the distribution and density of magnetic flux across the membrane can be changed when the cell is exposed to an electromagnetic field. Consequently, the electrical activities of the biological neuron can be changed due to the electromagnetic induction during the exchange of ion currents and the fluctuation of ion concentrations. Therefore, the effects of electromagnetic radiation on the dynamics of the nervous system should be considered with the increasing use of electric devices. As we all know, the magnetic (voltage)-controlled memristor describes the relation between magnetic flux and its memductance. When the effect of electromagnetic radiation on a neuron is considered as magnetic flux across the membrane of the neuron, the coupling between magnetic flux and membrane potential can be described by using a magnetic-controlled memristor [21,24]. As a consequence, the neuron under electromagnetic radiation can be modeled by adding an induction current in the traditional neuron model. The reduced diagram of the model of the neuron under electromagnetic radiation is given in Fig. 7. As shown in Fig. 7, the electromagnetic induction current can be computed by a memristor current, namely

$$I_{\mathrm{ER}} = kW(\varphi)V_{\mathrm{mem}} \tag{20}$$

where φ describes the magnetic flux across the membrane of a neuron, k denotes the coupling strength between membrane potential and magnetic flux. When

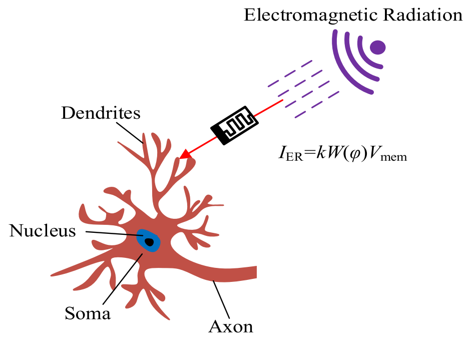

Fig. 7 Schematic diagram of the neuron under electromagnetic radiation

the electromagnetic induction current I_{ER} in Eq. (20) is considered in the traditional neuron model in Eq. (10), the model of the single neuron under electromagnetic radiation can be expressed by

$$\begin{cases} \mathrm{d}V_{\mathrm{mem}}/\mathrm{d}t = f(V_{\mathrm{mem}}, V_y, \ldots, I_{\mathrm{ext}}) - I_{\mathrm{ER}} \\ \mathrm{d}V_y/\mathrm{d}t = f(V_{\mathrm{mem}}, V_y, \ldots) \\ \vdots \\ \mathrm{d}\varphi/\mathrm{d}t = \mu V_{\mathrm{mem}} - h(\varphi) \end{cases} \tag{21}$$

where I_{ER} defines the feedback current on membrane potential when the magnetic flux is changed. The terms μV_{mem} and $h(\varphi)$ are the membrane potential-induced changes on magnet flux and leakage of magnet flux, respectively. Based on the memristive neuron model under electromagnetic radiation, the influence of electromagnetic radiation on neuronal activity in a single neuron can be studied. For example, reference [108] shows that electromagnetic radiation can excite quiescent neurons but also can suppress the electrical activities in the neuron as well. Coexisting multiple firing patterns in single neuron are detected when external electromagnetic radiation is imposed on the neuron [109–113]. In [114,115], the phenomenon of hidden homogeneous extreme multistability is discovered in a memristive HR neuron under electromagnetic field. Furthermore chaotic behaviors are observed in the neuron subjected to external electromagnetic radiation [115–117].

4.5 Neural network under electromagnetic radiation

Inspired by the mechanism of a single neuron under electromagnetic radiation, the neural network with multiple neurons under electromagnetic radiation can be modeled. The conceptual structure of the neural network exposed to electromagnetic radiation is shown in Fig. 8. In Fig. 8, when the neuron i (N_i) in the neural network is subjected to electromagnetic radiation, the induction current $I_{\mathrm{ER}i}$ will be induced by electromagnetic radiation. And the induction current can be described by $kW(\varphi)v_i$, where k is the feedback gain of the external stimulus induced by electromagnetic radiation, $W(\varphi)$ is the memductance of the memristor, and v_i is the membrane potential of the neuron i.

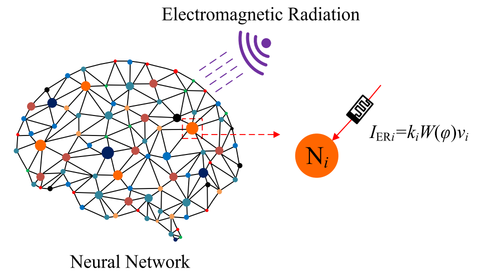

Fig. 8 Schematic diagram of the neural network under electromagnetic radiation

According to the above analysis, assuming that the Hopfield neural network is exposed to electromagnetic radiation, a Hopfield neural network under electromagnetic radiation can be modeled by [36]

$$\begin{cases} C_i \mathrm{d}x_i/\mathrm{d}t = -x_i/R_i + \sum_{j=1}^{n} w_{ij}v_j + I_{\mathrm{ER}i} + I_{i\mathrm{ext}} \\ \mathrm{d}\varphi_i/\mathrm{d}t = \mu_i x_i - h(\varphi_i) \end{cases} \tag{22}$$

where $I_{\mathrm{ER}i} = k_i W(\varphi_i)x_i$, φ_i is the magnetic flux across the membrane of neuron i, μ_i denotes the contribution of magnetic flux on the formation of neuron i membrane potential. The term $h(\varphi_i)$ represents the leakage of the magnetic flux. Based on this model, the influence of electromagnetic radiation on chaotic neurodynamics in neural networks can be analyzed. For example, the complex coexistence of periodic attractors, chaotic attractors, and transient chaotic attractors has been observed in a small HNN under electromagnetic radiation [27]. Hidden extreme multistability with hyperchaos and transient chaos is discussed in a three neuron-based HNN under electromagnetic radiation [35]. Furthermore, some coupled bi-neuron networks under electromagnetic radiation have been studied based on this mechanism [118–121]. And some chaotic dynamics such as chaos, chaotic bursting, and coexisting attractors are reported in these neural networks.

Table 1 Comparison of different neurons and neural networks

References	Periodic spiking	Periodic bursting	Chaotic spiking	Chaotic bursting	Chaos	Coexisting attractors	Hidden attractors	Multistability	Extreme multistability
HR [13]	Yes	Yes	No	No	No	No	No	No	No
FHN [76]	Yes	No	No	No	No	No	No	No	No
ML [6]	Yes	Yes	No	No	No	No	No	No	No
Chay [14]	No	Yes	No	No	No	No	No	No	No
HNN [9]	No	No	No	No	Yes	No	No	No	No
CNN [82]	No	No	No	No	No	No	No	No	No
M-HR [26]	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No
M-HR [97]	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No
M-FHN [98]	Yes	Yes	Yes	Yes	Yes	No	No	No	No
M-FHN [52]	Yes	No	No	No	Yes	Yes	Yes	Yes	Yes
M-ML [102]	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes
M-ML [120]	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No
M-Chay [101]	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No
M-HNN [35]	No	No	No	No	Yes	Yes	Yes	Yes	Yes
M-HNN [46]	No	No	No	No	Yes	Yes	No	Yes	Yes
M-CNN [50]	No	No	No	No	Yes	Yes	Yes	Yes	No
M-CNN [55]	No	No	No	No	Yes	Yes	No	No	No

To better exhibit the chaotic dynamics of the memristive neurons and neural networks, we give a performance comparison with the original neurons and neural networks, as shown in Table 1. As can be seen, the original neurons and neural networks only generate some simple dynamical behaviors such as periodic spiking, periodic bursting, and simple chaos. On the contrary, the memristive neurons can generate multiple firing patterns including periodic and chaotic spiking firings, periodic and chaotic bursting firings, as well as complex chaos. Moreover, the memristive neural networks exhibit more complex dynamics, especially chaotic dynamics including chaos, coexisting attractors, hidden attractors, multistability, and extreme multistability. Consequently, the study of memristive neurons and neural networks is significantly important to promote the development of computational neurodynamics.

5 Open problems and future work

As reviewed above, the chaotic dynamics of the memristive neuron and neural network models have greatly stimulated researchers' interest and many valuable research results have been reported until now. However, several important questions still remain to be answered. As we all know, the nervous system has a large number neurons and these neurons have different biological structures and functions. Whereas most memristive neuron and neural network models consider only a special structure and a single biological function. Thus, more different memristive neuron and neural network models need to be developed based on the different biological neuronal systems, such as astrocytes [122], microglia [123], and so on [124]. Moreover, the biological neurons and nervous systems are very sensitive to their living environment. At present, the ubiquitous factors including light, sound, and temperature are rarely considered in the existing memristive neuron and neural network models. In fact, building a reliable and multifunctional neuron model is critical for estimating complex neurodynamics. Besides, from the viewpoint of biophysical mechanism and function, the physical effects of electromagnetic induction should be considered in the neurons and neural networks [58,125]. Indeed, Ma et al [126–128] regard that the biophysical effects, biological function, and field coupling between neurons and neuronal networks should be considered. Although several interesting results related to physical effects in memristive neurons and neuronal networks have been revealed [129,130], more investigations need to do in further. From the viewpoint of application, the memristive neuron and neural network models can be used in various artificial intelligence fields including synchronous control [131–134], image encryption [135–137], and neuromorphic systems [138–140]. However, the relationship between the effects of these applications and the neurodynamics of the memristive neural systems is still unclear. That is to say, whether the influences of the external stimuli like electromagnetic radiation on the memristive neural systems in actual applications have the same effects as the biological nervous systems is an open problem. This would be a new area for exploring memristive neurodynamics.

6 Conclusion

Recent advances in neurodynamics by various memristive neuron and neural network models are discussed in detail which will certainly help the researchers to study this new prospect. Starting with the basics definition of chaotic dynamics, it slowly introduced the basics of the traditional artificial neuron and neural network models. Five types of memristive neuron and neural network models are summarized and addressed based on different biological neural mechanisms. It may be noted that the memristor plays an important and vital role in the memristive neuron and neural network models. Research results as reported by previous authors show that the memristive neural models have complex chaotic dynamics. Although some memristive neuron and neural network models and their chaotic dynamics have been investigated, it is still in the infant stage and the same need to be further explored. We believe that investigating the chaotic dynamics of memristive neuron and neural network models will help to elucidate more detailed functions of the brain as well as engineering applications.

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Declarations

Conflict of interest The authors declare that they have no conflicts of interest.

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