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A novel four-wing non-equilibrium chaotic system and its circuit implementation

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Abstract. In this paper, we construct a novel, 4D smooth autonomous system. Compared to the existing chaotic systems, the most attractive point is that this system does not display any equilibria, but can still exhibit four-wing chaotic attractors. The proposed system is investigated through numerical simulations and analyses including time phase portraits, Lyapunov exponents, bifurcation diagram, and Poincaré maps. There is little difference between this chaotic system without equilibria and other chaotic systems with equilibria shown by phase portraits and Lyapunov exponents. But the bifurcation diagram shows that the chaotic systems without equilibria do not have characteristics such as pitchfork bifurcation, Hopf bifurcation etc. which are common to the normal chaotic systems. The Poincaré maps show that this system is a four-wing chaotic system with more complicated dynamics. Moreover, the physical existence of the four-wing chaotic attractor without equilibria is verified by an electronic circuit.

Keywords. Four-wing; non-equilibrium; hidden attractor; Poincaré maps; circuit implementation.

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1. Introduction

Chaos theory has greatly developed in the past 40 years since Lorenz found the chaotic system in 1963 [1]. Many widely-known chaotic systems have been discovered, such as Lorenz [1], Rössler [2], Chua [3], Chen [4] and some other multiwing chaotic systems such as three wings and/or four wings etc. [5,6]. These classical attractors are excited from unstable equilibria. The equilibrium is very important for showing chaotic attractors, especially for showing multiple wings or scrolls.

Recent studies have involved classifying chaotic attractors as either self-excited attractor or hidden attractor [7–11]. A self-excited attractor has a domain of attraction that is associated with unstable equilibrium. The equilibrium plays many important roles. The central analytic criteria for the occurrence of chaos in these conventional chaotic systems are based on Shil'nikov [12,13] and Silva [14] methods. However, a hidden attractor has a domain of attraction that is not associated with any equilibrium points. Thus, chaotic flow with no equilibrium or with only stable equilibrium is bound to have a hidden attractor. Hidden attractors have important engineering applications because they allow unexpected and potentially disastrous responses to perturbations in a structure like a bridge or an airplane wing. Some examples of hidden attractors with only stable equilibria, such as modified Chua systems [8,9], a 3D autonomous quadratic system with only one stable equilibrium [15], a 3D chaotic system with only two stable node-foci [16], a generalized Sprott C system with two stable equilibria [17], have been reported. Several examples of a hidden attractor with no equilibria have also been reported in the literature, such as the perturbed Sprott D system with no equilibria [18], a hidden hyperchaotic attractor with no equilibria [19], fractional-order chaotic systems without equilibrium point [10,20]. However, all of these hidden attractors with no equilibria can only generate two-wing chaotic attractor at most. To our knowledge, there is no report about the existence of four-wing chaotic attractor without equilibrium point.

Based on these considerations, we construct a novel 4D smooth autonomous system. Compared with the existing chaotic systems, the most attractive point is that the model does not display any equilibria, but can exhibit four-wing chaotic attractor. The proposed system is investigated using numerical simulations and analyses including time-phase portraits, Lyapunov exponents, bifurcation diagram and Poincaré maps. The physical existence of the four-wing chaotic attractor without equilibria is verified by an electronic circuit.

This paper is organized as follows. In §2, the proposed dynamic system is studied by numerical and theoretical analyses. In §3, the oscillator circuit is designed for implementation. In §4, we draw our conclusions.

2. The proposed 4D dynamical system

The model to be investigated in this paper can be described as follows:

$$\begin{cases} \dot{x} = a_1 x - yz - \mathrm{d}_1, \\ \dot{y} = -b_1 y + xz + w, \\ \dot{z} = -c_1 z + xy, \\ \dot{w} = -e_1 x. \end{cases} \tag{1}$$

Here x, y, z, w are state variables, a_1, b_1, c_1, d_1 and e_1 are known non-negative constants. If there are equilibria for system (1), they can be obtained by solving: $\dot{x} = 0$, $\dot{y} = 0$, $\dot{z} = 0$ and $\dot{w} = 0$. But the equations have no solution. Hence, in system (1), there is no equilibrium.

2.1 *Four-wing chaotic system without equilibria*

In this paper, the numerical simulations are carried out using MATLAB. The fourth-order Runge–Kutta integration algorithm was performed to solve differential equations. The initial condition is set to $[1, 1, 1, 1]^T$. The new system can display a four-wing chaotic attractor if the parameters are properly chosen. Setting the parameters $(a_1, b_1, c_1, d_1, e_1) =$

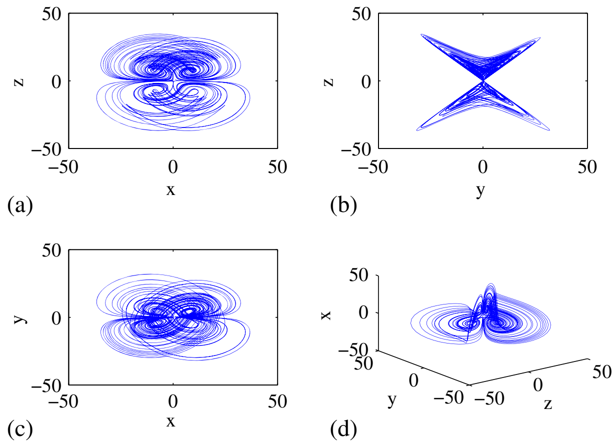

Figure 1. Phase portraits of the four-wing chaotic attractor: (**a**) x–z plane; (**b**) y–z plane; (**c**) x–y plane; (**d**) x–y–z space.

(4, 12, 5.5, 10, 0.5), as seen in figure 1, the system has generated a four-wing chaotic attractor.

2.2 *Lyapunov spectra*

When the parameters $(a_1, b_1, c_1, d_1) = (4, 12, 5.5, 10)$ are fixed, while parameter e_1 is varied, the spectra of maximum Lyapunov exponents are obtained as shown in figure 2. From figure 2, we see that the maximum Lyapunov exponent is positive, which implies that the system is chaotic.

Remark. The positive Lyapunov exponents are widely used nowadays as an indication of chaotic behaviour in a system. While this fact is widely used and is correct for most of the systems, in general case for a particular system it may be not true, e.g., the so-called Perron effects of Lyapunov exponents sign reversal. For rigorous use of Lyapunov

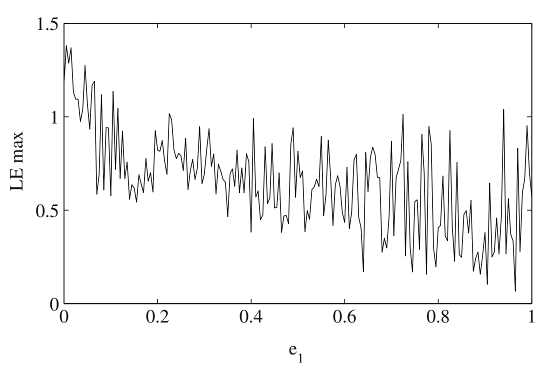

Figure 2. The spectrum of maximum Lyapunov exponents of system (1) with respect to e_1.

exponents, one has to check the conditions of Oseledets theorem (to prove ergodicity, etc.) which is a very difficult task [21].

2.3 *Bifurcation diagram and Poincaré map*

When the parameters $(a_1, b_1, c_1, d_1) = (4, 12, 5.5, 10)$ are fixed, while parameter e_1 is varied, the corresponding bifurcation diagram of state x with respect to e_1 is obtained as shown in figure 3. From figure 3, we see that there are no characteristics as the common chaotic systems with equilibia such as pitchfork bifurcation and Hopf bifurcation.

The Poincaré map can reflect bifurcation and folding properties of chaos. We have taken different cross-sections and the system parameters are set to $(a_1, b_1, c_1, d_1, e_1) = (4, 12, 5.5, 10, 0.5)$. Figure 4 shows projections of the Poincaré map on different cross-

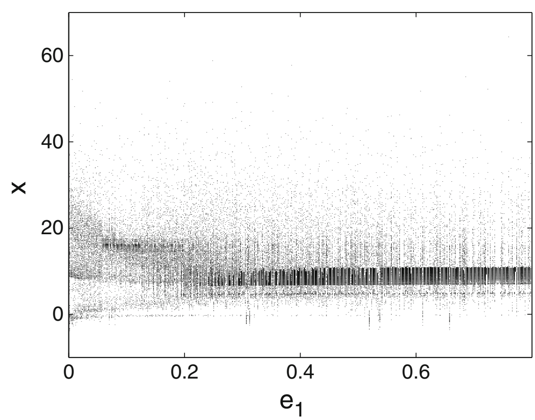

Figure 3. Bifurcation diagram of the state variable x vs. parameter e_1.

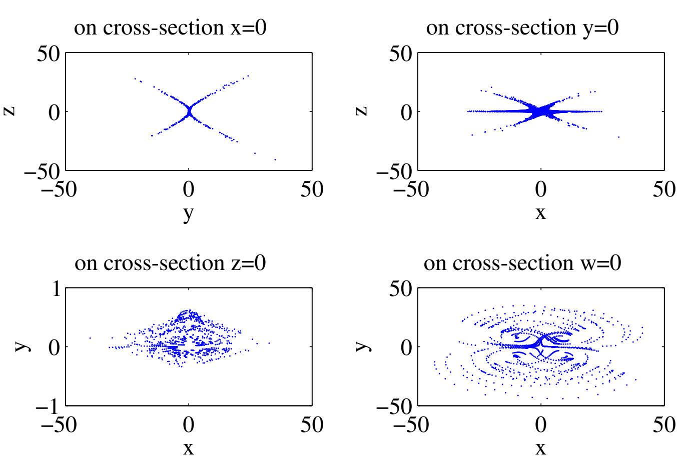

Figure 4. Poincaré maps of the four-wing hyperchaotic attractor with parameters $a_1 = 4$, $b_1 = 12$, $c_1 = 5.5$, $d_1 = 10$, $e_1 = 0.5$, on different cross-sections $x = 0$, $y = 0$, $z = 0$, $w = 0$.

sections: $x = 0$; $y = 0$; $z = 0$; $w = 0$, one can see that the Poincaré map here consists of several limbs with various bifurcations in different directions, which indicates that the system has extremely rich dynamics. Also, the Poincaré maps show that the branches are joined and united as a single attractor. This proves the existence of four-wing chaotic attractor with more complicated dynamics.

3. Circuit implementation

In this section, an electronic circuit is designed to realize system (1). According to figure 1, the maximum of state variables is 100. To prevent the operational amplifiers and analogue multipliers from saturation, the linear transformation $x = 10x_2$, $y = 10y_2$, $z = 10z_2$ and $w = 10w_2$ can be used and the minification is 10. System (1) with $(a_1, b_1, c_1, d_1, e_1) = (4, 12, 5.5, 10, 0.5)$ will be changed to

$$\begin{cases} \dot{x} = a_2x_2 - y_2z_2 - d_2, \\ \dot{y} = -b_2y_2 + x_2z_2 + w_2, \\ \dot{z} = -c_2z_2 + x_2y_2, \\ \dot{w} = -e_2x_2, \end{cases} \tag{2}$$

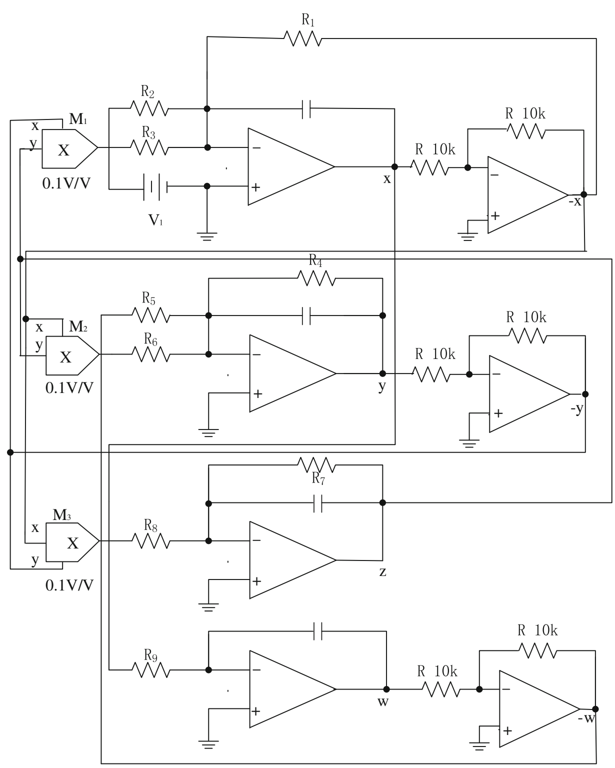

Figure 5. Circuit diagram to realize the chaotic attractor of the novel system.

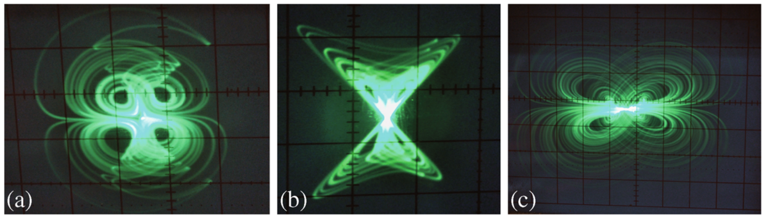

Figure 6. Experimental observations of the chaotic attractor: (**a**) x–z plane (1 V/Div, 1 V/Div), (**b**) y–z plane (1 V/Div, 2 V/Div), (**c**) x–y plane (1 V/Div, 1 V/Div).

where $(a_2, b_2, c_2, d_2, e_2) = (4, 12, 5.5, 1, 0.5)$. Clearly, since the linear revertible transform does not change the structure of the state space, such an amplitude reduction keeps the pattern of the attractor. The designed circuit realizing (2) is presented in figure 5. The resistors in figure 5 are chosen as $R_3 = R_6 = R_8 = 10\,\mathrm{K\Omega}$, $R_1 = 250\,\mathrm{K\Omega}$, $R_2 = 100\,\mathrm{M\Omega}$, $R_4 = 83.3\,\mathrm{K\Omega}$, $R_5 = 100\,\mathrm{K\Omega}$, $R_7 = 181.8\,\mathrm{K\Omega}$, $R_9 = 2\mathrm{M\Omega}$. The voltage is chosen as $V_1 = 1\,\mathrm{V}$. The operational amplifiers are of type LT082 and the multipliers are of type AD633 with an output coefficient of 0.1. The power is supplied by 15 V. The experimental observations from the analogue oscilloscope are shown in figure 6. This experiment shows that the system with the above-mentioned parameters can generate a real four-wing chaotic attractor. Comparing the numerical simulation and the circuit experimental results, it can be declared that a very good qualitative agreement between the two parts has been confirmed.

4. Conclusion

In this paper, a novel four-wing non-equilibrium chaotic system is introduced. The dynamic behaviour is analysed and the presence of four-wing chaotic attractor in the absence of equilibria is validated. Besides that, an oscillator circuit is designed to verify that the system without equilibrium point can generate a four-wing chaotic attractor. On account of the complicated topological structures, the proposed four-wing chaotic system without equilibrium may be further studied theoretically and has a promising application in the field of information technology such as secure communication and encryption.

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