A Novel Adaptive Active Control Projective Synchronization of Chaotic Systems

Article · February 2018

DOI: 10.1115/1.4039189

A Novel Adaptive Active Control Projective Synchronization of Chaotic Systems

Boan Quan
School of Information Science and Engineering,
Hunan University,
Changsha 410082, China
e-mail: quanboan@163.com

Chunhua Wang[1]
School of Information Science and Engineering,
Hunan University,
Changsha 410082, China
e-mail: wch1227164@hnu.edu.cn

Jingru Sun
School of Information Science and Engineering,
Hunan University,
Changsha 410082, China
e-mail: jt_sunjr@hnu.edu.cn

Yilin Zhao
School of Information Science and Engineering,
Hunan University,
Changsha 410082, China
e-mail: zhaoyl1992@126.com

This paper investigates adaptive active control projective synchronization scheme. A general synchronization controller and parameter update laws are proposed to stabilize the error system for the identical structural chaotic systems. It is the first time that the active synchronization, the projective synchronization, and the adaptive synchronization are combined to achieve the synchronization of chaotic systems, which extend the control capability of achieving chaotic synchronization. By using a constant diagonal matrix, the active control is developed. Especially, when designing the controller, we just need to ensure that the diagonal elements of the diagonal matrix are less than or equal 0. So, the synchronization of chaotic systems can be realized more easily. Furthermore, by proposing an active controller, in combination with several different control schemes, we lower the complexity of the design process of the controller. More importantly, the larger the absolute value of product of the diagonal elements of diagonal matrix is, the smoother the curve of chaotic synchronization is and the shorter the time of chaotic synchronization is. In our paper, we take Lorenz system as an example to verify the effectiveness of the proposed synchronization scheme. Theoretical analysis and numerical simulations demonstrate the feasibility of this control method. [DOI: 10.1115/1.4039189]

Keywords: chaotic synchronization, active synchronization, projective synchronization, adaptive synchronization, active control

1 Introduction

Synchronization is one of the hot parts of chaotic areas. Since the seminal work of Pecora and Carroll [1], chaos synchronization has received a great deal of interest among scientists from various fields. Usually, when studying chaotic synchronization, we use external signals to drive the responding chaotic system, such as in Refs. [2] and [3]. Chaotic synchronization has been successfully applied in secure communication, information processing, life science, etc. [4–6] In the past years, many synchronization techniques, including phase synchronization [7], adaptive synchronization [8], lag synchronization [9], complete synchronization [10], coupling synchronization [11], fuzzy sliding mode control synchronization [12,13], etc. have been studied. All of the synchronization methods mentioned above have their own disadvantages. In phase synchronization method, one or more components of the error system cannot converge to 0 at last, such as in Ref. [14]. So, it will increase error between the drive system and response system. In adaptive synchronization method, the controller design and the parameter update laws design usually become very complex [15]. Therefore, it will increase the difficulty to control synchronization. In lag synchronization, the time delay of the state vector usually becomes difficult to control, such as in Ref. [16]. So, it will increase error between the drive system and response system. In complete synchronization, the synchronization scheme has no universal conclusion such as in Ref. [17]. In coupling synchronization, it is difficult to determine the coupling factor, such as in Ref. [18].

Recently, projective synchronization has attracted a great amount of attention [19–21] for its unpredictability of the scaling factor or the scaling function. It can additionally enhance the security of communication. In Ref. [15], Dibakar Ghosh proposed a projective scheme in multiple modulated time-delayed systems. However, in that paper, the author did not analyze the design of controller. In Ref. [22], Wang et al. proposed a high precision fast projective synchronization method. It increases the speed of synchronization compared to the general method. However, they did not do any research about parameter adaption. And the mathematical reasoning process is also complex. In Ref. [19], Wang et al. proposed a time-controllable projective synchronization scheme. In that paper, the synchronization method can improve the speed of synchronization by adjusting some parameters. Meanwhile, the synchronization scheme can improve the ability of antideciphering information because of its unknown scaling matrix and unknown convergence. However, the designing process of the controller and the parameter update laws is also complex. In Ref. [20], Li et al. proposed a unified method for projective synchronization of chaotic system. In that paper, a versatile model of chaotic projective synchronization was proposed by constructing a generalized proportion matrix. So, all kinds of projective synchronization schemes could be achieved by varying the generalized proportion matrix. And because of these characteristics, this synchronization scheme extended the applicability of projective synchronization. However, the response system of the paper was constructed artificially. Therefore, they were not suitable to the application of arbitrary response system. Moreover, they did not also consider the case of parameter adaptation. In Ref. [21], Li et al. proposed a new projective scheme. In their paper, they put forward a novel design scheme of controller. It is independent of the Routh–Hurwitz criterion. That meant the complexity of active control was simplified. However, the paper did not consider the case of parameter adaption. In conclusion, for the above mentioned papers, most of the controller design of them is complex. Therefore, the synchronization between the drive system and the response system is difficult to be controlled. Meanwhile, the tunability of the synchronization scheme is poor. Moreover, almost none of the papers mentioned above did study how to shorten the time of synchronization and make the curves of chaotic synchronization smoother.

[1]Corresponding author.

Contributed by the Design Engineering Division of ASME for publication in the Journal of Computational and Nonlinear Dynamics. Manuscript received May 17, 2017; final manuscript received January 20, 2018; published online March 23, 2018. Assoc. Editor: Bogdan I. Epureanu.

To overcome the weaknesses above, we propose an adaptive active control projective scheme and the scheme of shortening synchronized time and making the curves of chaotic synchronization smoother than the ones in other synchronization methods. Compared with the articles above, the proposed control scheme has the following advantages. First, we proposed a general synchronization controlling scheme and parameters update laws. It extends the scope of application of the synchronization control scheme and parameters update laws. Second, a new method of combining the active control, the adaptive control with projective control is proposed. This synchronization scheme pioneered the mutual combination of these three methods. Therefore, it has extended the scope of application of the synchronization scheme. Third, the proposed synchronization scheme can make chaotic synchronization easier to be realized by adjusting the constant diagonal matrix. Last, but not least, by using different constant diagonal matrix in our paper, the synchronization between the drive system and the response system can also be achieved. That means that we have extended the adjustable range of synchronization of chaotic system. Meanwhile, we can shorten the time of synchronization and make the curves of chaotic synchronization smoother than the ones in other synchronization methods by enlarging the absolute value of product of the diagonal elements of diagonal matrix C.

This paper is organized as follows: In Sec. 2, we introduce our analysis of our synchronization principle and prove it. In Sec. 3, we take Lorenz system as an example to verify effectiveness of the proposed synchronization scheme. Meanwhile, we take different constant diagonal matrix C to indicate that we have extended the adjustable range of synchronization of chaotic system. In Sec. 4, we analyze the advantages of our synchronization method in detail and give comparative results as well. Finally, the conclusions are drawn in Sec. 5.

2 Analysis of Synchronization Principle

2.1 The General Mathematical Model of Synchronization of Chaotic Systems. In Ref. [5], the mathematical model of the drive system and the response can be described as follows:

$$\dot{x} = f(x) + F(x)\theta \tag{1}$$

$$\dot{y} = f(y) + F(y)\breve{\theta} + u \tag{2}$$

$$u = -\breve{f} - \breve{F}\bar{\theta} + Ae \tag{3}$$

where $x, y \in R^n$ are the state variables of the drive and response systems, respectively, $f : R^n \to R^n$ are the continuous vector function and, $F \in R^{n\times p}$ is the function matrix, $\theta \epsilon R^p$ is an unknown parameter vector, $\bar{\theta} = \breve{\theta} - \theta$ represents the estimate vector of unknown parameter vector θ, $u \epsilon R^p$ is controller to be determined, $A\in R^n$ is the coefficient matrix of the vector error state e, $\breve{f} = f(y) - Hf(x)$, $\breve{F} = F(y) - HF(x)$, H is a n-order diagonal matrix. A is chosen such that it has all its eigenvalues on the left-hand side of the complex plane. To make the drive system (1) and the response system (2) realize adaptive projective synchronization, there must exist a positive symmetric matrix P such that

$$A^{\mathrm{T}}P + PA = -Q \tag{4}$$

where Q denotes a positive symmetric matrix. Meanwhile, A is chosen such that it has all its eigenvalues on the left-hand side of the complex plane. However, the two above-mentioned conditions are not easy to be realized. Therefore, it makes the controller not flexible to be adjusted. That is to say, it makes the synchronization between the drive system (1) and the response system (2) not easy to be realized. To overcome weaknesses above mentioned, in Sec. 2.2, we propose the active synchronization scheme. According to the active synchronization scheme, we just ensure the diagonal elements of the diagonal matrix less than or equal **0**. The design of the controller is very easy. Meanwhile, the controller is flexible to be adjusted.

2.2 Proposed Mathematical Model of Synchronization. The proposed mathematical model of drive system and response system in our paper can be described as follows:

$$\dot{X} = f(X) + F(X)\theta + AX \tag{5}$$

$$\dot{Y} = f(Y) + F(Y)\breve{\theta} + BY + U \tag{6}$$

where X and $Y \in R^n$ are the state vectors, $f : R^n \to R^n$ is continuous nonlinear vector functions $U \in R^n$ is the vector controller, $F \in R^{n\times n}$ is the function matrix, $A, B \in R^{n\times n}$ is the constant matrix, $\theta \epsilon R^n$ is the control parameter vector, and $\breve{\theta} \in R^n$ is the unknown parameter vector that needs to be distinguished during the process of the synchronization of the drive-response system. In fact, Eq. (5) can be changed into Eq. (1). Meanwhile, Eq. (6) can be changed into Eq. (2). In Eq. (5), AX and $f(X)$ denote linear part and nonlinear part of $f(x)$ in Eq. (1), and $F(X)\theta$ represents $F(x)\theta$ in Eq. (1). Similarly, in Eq. (6), BY and $f(Y)$ denote linear part and nonlinear part of $f(y)$ in Eq. (2), and $F(Y)\theta$ represents $F(y)\theta$ in Eq. (2), and U represents u in Eq. (2). However, the controller U and $\breve{\theta}$, whose expressions in Eq. (6) are unlike as the ones of u and $\breve{\theta}$ in Eq. (2), are to be designed in this paper.

Define the error system

$$e(t) = Y - \alpha X \tag{7}$$

where $\alpha \in R$ is the scaling factor, which is a real constant.

Our goal is to design the active controller and the update law $\breve{\theta}$such that the controlled response system (6) could be adaptive projective synchronization to the drive system (5).

2.3 Design of Active Controller and Parameter Update Law. In this section, we introduce the design scheme of controller and the parameter update laws. Compared with the synchronization schemes in Sec. 2.2, we propose a novel adaptive active control projective synchronization. By using active control scheme, we introduce a constant diagonal matrix C. On one hand, the synchronization of the chaotic system is easier to achieve because we just need to adjust the C that ensures the diagonal elements of the diagonal matrix are less than or equal **0**. Therefore, the synchronization between the drive system and the response system is more flexible to be achieved. On the other hand, the controller in our paper is universal. Therefore, it extended the scope of application of the synchronization control scheme and parameters update laws.

Definition 1. *In Eq. (7), if there exists a real constant $\alpha \in R$ such that $\lim_{t\to\infty} \|e\| = 0$, then we regard that the system (5) and the system (6) are synchronized, which is called "generalized projective synchronization."*

Definition 2. *V(x) is a scalar function, where x is the system state vector, and if V(x) has the following properties: (1) V(x) is a continuous function (2) V(x) is a positive function (3) When $\|X\| \to \infty$, $\|V\| \to \infty$. Then V(x) is Lyapunov function.*

Lemma 1 *$\dot{V}(x)$ is negative semidefinite in the neighborhood of the equilibrium point, and with the movement of the system state, $\dot{V}(x)$ is less than or equal to 0, then the system is stable at the equilibrium point. $\dot{V}(x)$ is the derivative of V(x), Ref. [23].*

Theorem 1. *For given a projective scaling factorα, and any initial conditions X(0), Y(0),$\breve{\theta}(0)$, the drive system (5) and the response system (6) can realize active adaptive projective synchronization with the active controller (10) and the parameter update law (11) as below:*

$$h(X, Y, \alpha, \theta) = f(Y) + BY - \alpha F(X)\theta - \alpha AX \tag{8}$$

$$V(t) = \mathrm{Ce} \quad (9)$$

$$U = -h(X, Y, \alpha, \theta) + V(t) - F(Y)\theta \quad (10)$$

$$\breve{\theta}^{\cdot} = -(F(Y))^{\mathrm{T}} e \quad (11)$$

where $\breve{\theta}^{\cdot}$ represents the time derivative of $\breve{\theta}$, C is a constant diagonal matrix, and the diagonal elements of C is less than or equal 0.

Proof. The time derivative of the error system (7) is

$$\dot{e}(t) = \dot{Y} - \alpha\dot{X} \quad (12)$$

Substituting Eqs. (5) and (6) into Eq. (12), we have

$$\dot{e}(t) = F(Y)\breve{\theta} + f(Y) + BY + U - \alpha F(X)\theta - \alpha f(X) - \alpha AX \quad (13)$$

Substituting Eq. (8) into Eq. (13), we have

$$\dot{e}(t) = F(Y)\breve{\theta} + H(X, Y, \alpha, \theta) + U \quad (14)$$

Substituting Eq. (10) into Eq. (14), we have

$$\dot{e}(t) = F(Y)\breve{\theta} + V(t) - F(Y)\theta \quad (15)$$

Let $\bar{\theta} = \breve{\theta} - \theta$, we have

$$\dot{e}(t) = F(Y)\bar{\theta} + V(t) \quad (16)$$

We proceed next with a quadratic Lyapunov function candidate for ensuring asymptotic stability of the error system (7) as follows:

$$w(t) = \frac{1}{2}(\bar{\theta})^{\mathrm{T}}(\bar{\theta}) + \frac{1}{2}e^{\mathrm{T}}e \quad (17)$$

The time derivative of the $w(t)$ is

$$\dot{w}(t) = \frac{1}{2}\left[(\bar{\theta})^{\cdot\mathrm{T}}(\bar{\theta}) + (\bar{\theta})^{\mathrm{T}}\bar{\theta}^{\cdot}\right] + \frac{1}{2}\left[(\dot{e})^{\mathrm{T}}e + e^{\mathrm{T}}\dot{e}\right] \quad (18)$$

where $(\bar{\theta})^{\cdot}$ represents the time derivative of $-\theta$, and $\dot{e}$ represents the time derivative of e and the constant diagonal matrix $C = \mathrm{diag}\{k_1, k_2, \ldots, k_n\}$, $e = [e_1 e_2 \ldots e_n]^{\mathrm{T}}$.

Substituting Eqs. (15) and (11) into Eq. (18), we have

$$\begin{aligned}\dot{W}(t) &= \frac{1}{2}e^{\mathrm{T}}\mathrm{Ce} + \frac{1}{2}e^{\mathrm{T}}\mathrm{Ce} \\ &= e^{\mathrm{T}}\mathrm{Ce} \\ &= [e_1 e_2 \ldots e_n]\mathrm{diag}\{k_1, k_2, \ldots, k_n\}[e_1 e_2 \ldots e_n]^{\mathrm{T}} \\ &= \sum_{i=0}^{n} k_i e_i^2 \le 0\end{aligned} \quad (19)$$

According to Lemma 1, the error system (7) is asymptotic stable, so the error system (7) achieves asymptotic stability under the chosen active controller (10) and the parameter update law (11). This completes the proof, and that is to say, the drive system (5) and the response system (6) realize active adaptive projective synchronization.

3 Synchronization Simulations

In this section, we will take Lorenz system as an example to verify the effectiveness of the proposed synchronization scheme and take different values of matrix C to verify the tunability and flexibility of the chaotic synchronization. All of the simulations are carried out using MATLAB software and the Runge–Kutta method. Numerical simulations are performed to demonstrate the effectiveness of the proposed method.

3.1 System Description. The Lorenz system was proposed by the mathematician Lorenz [24], which can be described as follows:

$$\begin{cases}\dot{x} = a(y - x) \\ \dot{y} = cx - xz - y \\ \dot{z} = -xy - bz\end{cases} \quad (20)$$

Now, we replace b with c, and c with b, and we have

$$\begin{cases}\dot{x} = a(y - x) \\ \dot{y} = bx - xz - y \\ \dot{z} = xy - cz\end{cases} \quad (21)$$

Then we choose the drive system as follows:

$$\begin{cases}\dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = bx_1 - x_2 - x_1x_3 \\ \dot{x}_3 = x_1x_2 - cx_3\end{cases} \quad (22)$$

The response system is given by

$$\begin{cases}\dot{y}_1 = a_1(y_2 - y_1) + u_1 \\ \dot{y}_2 = b_1y_1 - y_2 - y_1y_3 + u_2 \\ \dot{y}_3 = y_1y_2 - c_1y_3 + u_3\end{cases} \quad (23)$$

The error system is described as below:

$$\begin{cases}e_1 = y_1 - \alpha x_1 \\ e_2 = y_2 - \alpha x_2 \\ e_3 = y_3 - \alpha x_3\end{cases} \quad (24)$$

Comparing Eq. (22) with Eq. (5), and Eq. (23) with Eq. (6), we have

$$F(X) = \begin{bmatrix} x_2 - x_1 & 0 & 0 \\ 0 & -x_1 & 0 \\ 0 & 0 & -x_3 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$f(X) = \begin{bmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{bmatrix} \quad \theta = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (25)$$

Comparing Eq. (22) with Eq. (5), and Eq. (23) with Eq. (6), we have

$$F(Y) = \begin{bmatrix} y_2 - y_1 & 0 & 0 \\ 0 & -y_1 & 0 \\ 0 & 0 & -y_3 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$f(Y) = \begin{bmatrix} 0 \\ -y_1y_3 \\ y_1y_2 \end{bmatrix} \quad \breve{\theta} = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \quad U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (26)$$

And let $C = \mathrm{diag}\{k_1, k_2, k_3\}$.

3.2 The Design of Adaptive Active Projective Controller. According to Theorem 1, we choose the controller laws and the parameter update law as below:

$$
\begin{cases}
u_1 = \alpha a(x_2 - x_1) + k_1 e_1 - a e_1 \\
u_2 = b a x_1 - \alpha x_1 x_3 - \alpha x_2 + y_1 y_2 + y_2 + k_2 e_2 - b y_1 \\
u_3 = -\alpha c x_3 + \alpha x_1 x_2 - y_1 y_2 + k_3 e_3 + c y_3
\end{cases} \tag{27}
$$

$$
\begin{cases}
\dot{a}_1 = (y_1 - y_2) e_1 \\
\dot{b}_1 = -y_1 e_2 \\
\dot{c}_1 = y_3 e_3
\end{cases} \tag{28}
$$

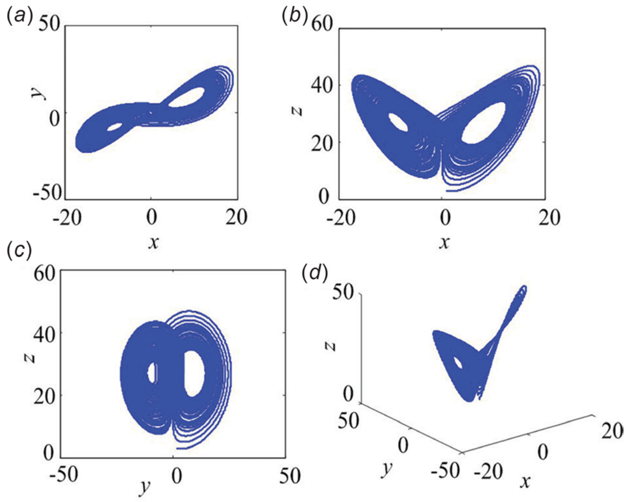

Fig. 1 The phase diagram of system (21): (*a*) the projection of the attractor of *x*–*y* plane, and (*b*) the projection of the attractor of *x*-*z* plane, and (*c*) the projection of the attractor of *y*–*z* plane, and (*d*) The projection of the attractor of *x*-*y*-*z* plane

3.3 Phase Diagram. When $a = 10$, $b = 28$, $c = 8/3$, the system (21) is chaotic, and the phase diagrams are shown in Figs. 1(*a*)–1(*d*).

3.4 The Verification of the Tunability of the Chaotic Synchronization. In this section, we take several different matrices C to verify the tenability of the flexibility of the chaotic synchronization. We set the scaling factor $\alpha = 2$. The initial values of the drive system (22) are set to be $x_1(0) = 1, x_2(0) = 2, x_3 = 3$, and the initial values of the response system (23) are set to be $y_1(0) = 4$, $y_2(0) = 5$, $y_3 = 6$, and the initial of the unknown parameters that needs to be distinguished are set to be $a_1(0) = 0.1$, $b_1(0) = 0.2$, $c_1 = 0.3$. Figures 2–4 show that the state trajectories of the response system asymptotically approach the drive system up to the given scaling factor. From Figs. 5–7, we can see

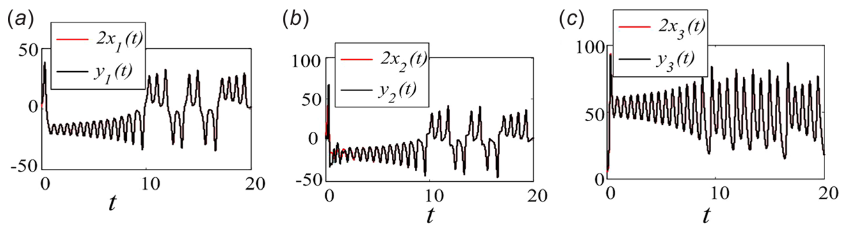

Fig. 2 The simulations of the synchronization between the drive system (22) and the response system (23) when $C = \mathrm{diag}\{-1, -2, -3\}$: (*a*) the synchronization between $y_1(t)$ and $2x_1(t)$, and (*b*) the synchronization between $y_2(t)$ and $2x_2(t)$, and (*c*) the synchronization between $y_3(t)$ and $2x_3(t)$

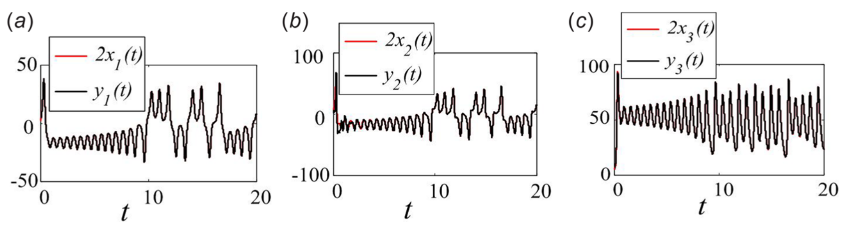

Fig. 3 The simulations of the synchronization between the drive system (22) and the response system (23) when $C = \mathrm{diag}\{-2, -2, -2\}$: (*a*) the synchronization between $y_1(t)$ and $2x_1(t)$, and (*b*) the synchronization between $y_2(t)$ and $2x_2(t)$, and (*c*) the synchronization between $y_3(t)$ and $2x_3(t)$

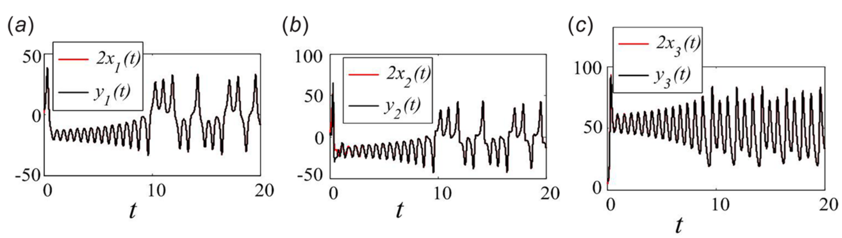

Fig. 4 The simulations of the synchronization between the drive system (22) and the response system (23) when $C = \mathrm{diag}\{-3, -3, -3\}$: (*a*) the synchronization between $y_1(t)$ and $2x_1(t)$, and (*b*) the synchronization between $y_2(t)$ and $2x_2(t)$, and (*c*) the synchronization between $y_3(t)$ and $2x_3(t)$

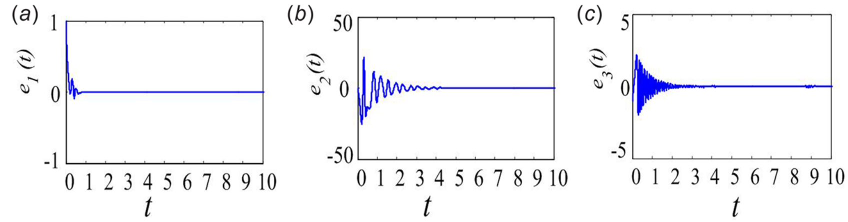

Fig. 5 The simulations of the errors between the drive system (22) and the response system (23) when $C = \mathrm{diag}\{-1, -2, -3\}$: (*a*) the evolution of e_1, (*b*) the evolution of e_2, and (*c*) the evolution of e_3

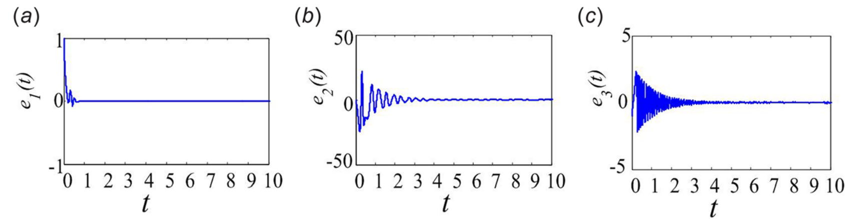

Fig. 6 The simulations of the errors between the drive system (22) and the response system (23) when $C = \mathrm{diag}\{-2, -2, -2\}$: (*a*) the evolution of e_1, (*b*) the evolution of e_2, and (*c*) the evolution of e_3

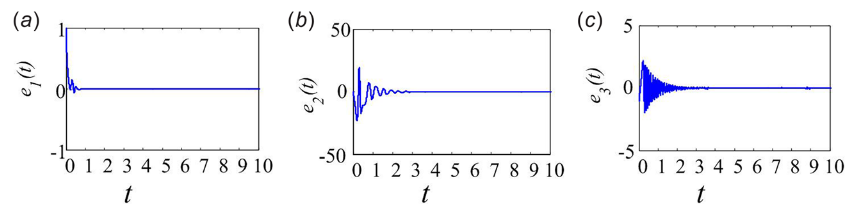

Fig. 7 The simulations of the errors between the drive system (22) and the response system (23) when $C = \mathrm{diag}\{-3, -3, -3\}$: (*a*) the evolution of e_1, (*b*) the evolution of e_2, and (*c*) the evolution of e_3

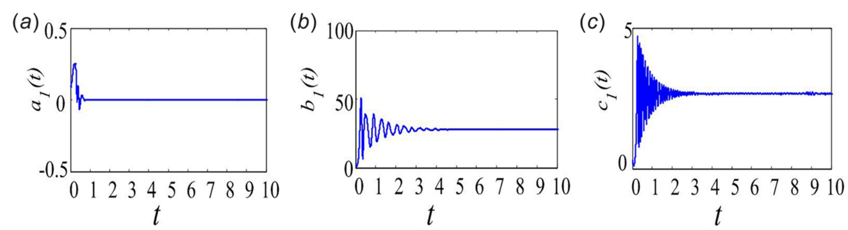

Fig. 8 The identifications of $a_1(t), b_1(t), c_1(t)$ when $C = \mathrm{diag}\{-1, -2, -3\}$: (*a*) the identification of $a_1(t)$, (*b*) the identification of $b_1(t)$, and (*c*) the identification of $c_1(t)$

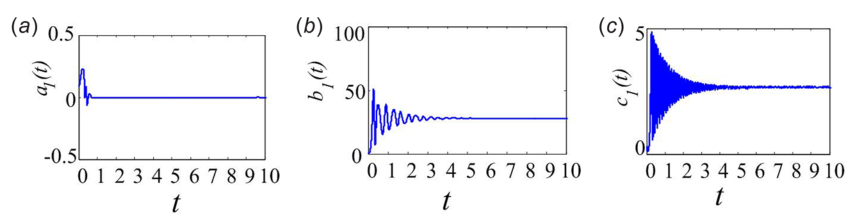

Fig. 9 The identifications of $a_1(t), b_1(t), c_1(t)$ when $C = \mathrm{diag}\{-2, -2, -2\}$: (*a*) the identification of $a_1(t)$, (*b*) the identification of $b_1(t)$, and (*c*) the identification of $c_1(t)$

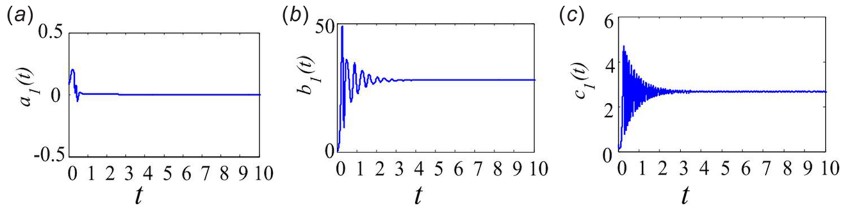

Fig. 10 The identifications of $a_1(t), b_1(t), c_1(t)$ when $C = \text{diag}\{-3, -3, -3\}$: (*a*) the identification of $a_1(t)$, (*b*) the identification of $b_1(t)$, and (*c*) the identification of $c_1(t)$

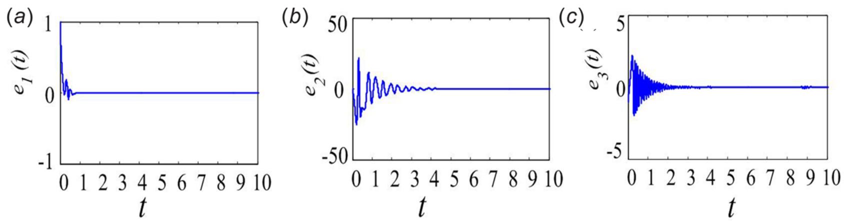

Fig. 11 The simulations of the errors between the drive system (22) and the response system (23) when $C = \text{diag}\{-1, -2, -3\}$, and the scaling factor $\alpha = 2$: (*a*) the evolution of, e_1, (*b*) the evolution of e_2, and (*c*) the evolution of e_3

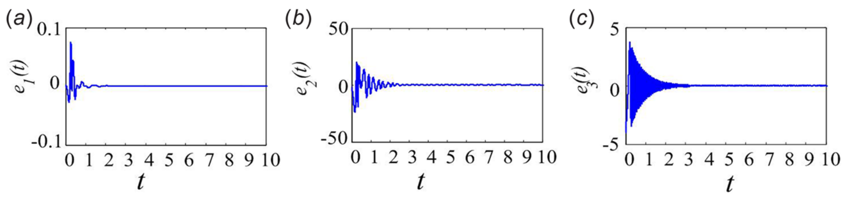

Fig. 12 The simulations of the errors between the drive system (22) and the response system (23) when $C = \text{diag}\{-1, -2, -3\}$, and the scaling factor $\alpha = 3$: (*a*) the evolution of, e_1, (*b*) the evolution of, e_2, and (*c*) the evolution of e_3

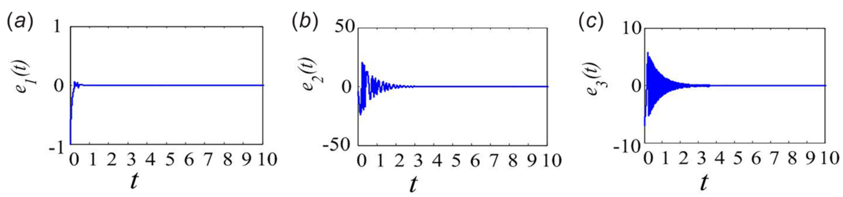

Fig. 13 The simulations of the errors between the drive system (22) and the response system (23) when $C = \text{diag}\{-1, -2, -3\}$, and the scaling factor $\alpha = 4$: (*a*) the evolution of e_1, (*b*) the evolution of, e_2, and (*c*) the evolution of e_3

that the errors indeed are close to 0. From Figs. 8–10, we can see that the unknown parameters of system (23), which need to be distinguished finally, adapt themselves to the true values by using the parameters update law (28). The results imply that the two identical chaotic systems realize adaptive active control projective synchronization. More importantly, in the above simulations, the synchronization between the drive system and the response system can be achieved by taking different constant matrix C. Therefore, the tunability of the chaotic synchronization is verified. In addition, the synchronization is easy to achieve because we just need to the diagonal elements of the diagonal matrix C less than or equal 0.

3.5 Simulations With Different Scaling Factor α. In this section, we take different scaling factor α to show that the synchronization of chaotic system can be achieved. We just take $\alpha = 2$, $\alpha = 3$, and $\alpha = 4$ for example for simulation in this paper.

Figures 11–13 show that the synchronization between the drive system and the response system can be achieved with different scaling factor α.

4 Discussion

In this section, we will introduce our advantages of synchronization in detail and then propose our scheme of controlling the time of chaotic synchronization and how to make the curve of chaotic synchronization smoother than the ones in other synchronization methods.

4.1 Our Advantages of Synchronized Method. Tahereh Binazadeh, etc. proposed adaptive synchronization in Ref. [25]. Hong-juan Liu, etc. proposed projective synchronization in Ref. [26]. Rong-An Tang proposed active control synchronization

Table 1 The approximate time of chaotic synchronization compared with other papers, and "—" denotes that the current error component does not exist

Papers	[28]	[29]	[30]	[31]	[32]	This paper
The time of e1 tending zero(s)	10	2.5	17	1	6	0.4
The time of e2 tending zero(s)	14	2.5	—	7	—	0.5
The time of e3 tending zero(s)	1	2.5	—	—	—	0.4

in Ref. [27]. However, all of them did not put forward a synchronization method combining these three methods. So, first, we present a combination of adaptive control, active control, and projective scheme for synchronization for the first time. Second, the synchronization method we present in our paper follows the logic order that is from general to specific. In our paper, from Eq. (5) to Eq. (19), we derive general equations about the synchronization scheme. And from Eq. (20) to Eq. (28), we take an example to illustrate our method. That is to say, we propose a general synchronization control scheme, which extends the scope of application of the synchronization control scheme and parameters update laws. Third, in Eq. (10), the constant matrix C is easy to adjust. We just need to ensure the diagonal elements of the diagonal matrix less than or equal 0.Then the synchronization of chaotic system can be achieved. Therefore, the synchronization of chaotic systems can be achieved more easily. More importantly, we can extend the adjustable range of synchronization of chaotic system

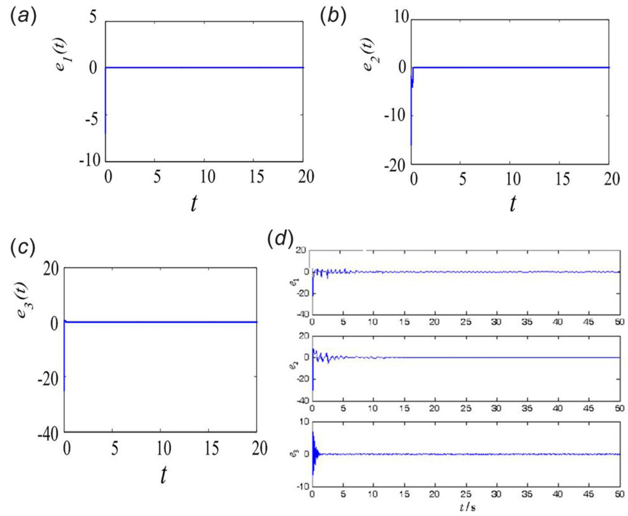

Fig. 17 **The simulations of the errors between the drive system (22) and the response system (23) when $C = \text{diag}\{-200, -200, -200\}$ and $\alpha = 10$: (*a*) the evolution of e_1, (*b*) the evolution of e_2, (*c*) the evolution of e_3, and (*d*) the simulations of the errors [28]**

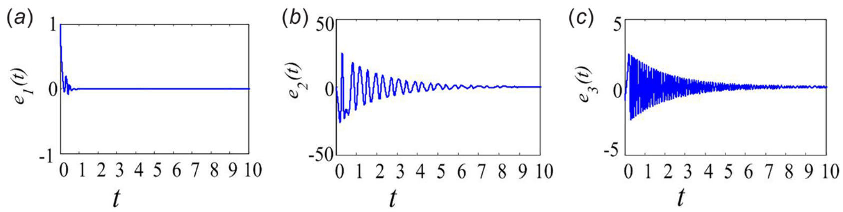

Fig. 14 **The simulations of the errors between the drive system (22) and the response system (23) when $C = \text{diag}\{-1, -1, -1\}$ and $\alpha = 2$: (*a*) the evolution of, e_1, (*b*) the evolution of, e_2, and (*c*) the evolution of e_3**

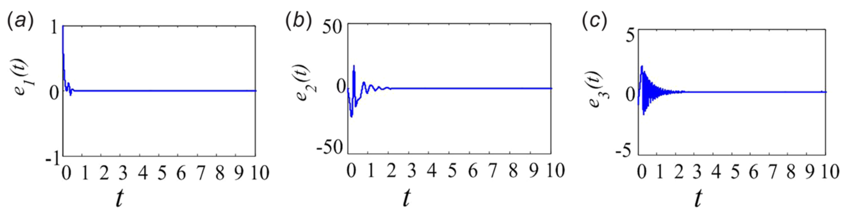

Fig. 15 **The simulations of the errors between the drive system (22) and the response system (23) when $C = \text{diag}\{-4, -4, -4\}$ and $\alpha = 2$: (*a*) the evolution of, e_1, (*b*) the evolution of e_2, and (*c*) the evolution of e_3**

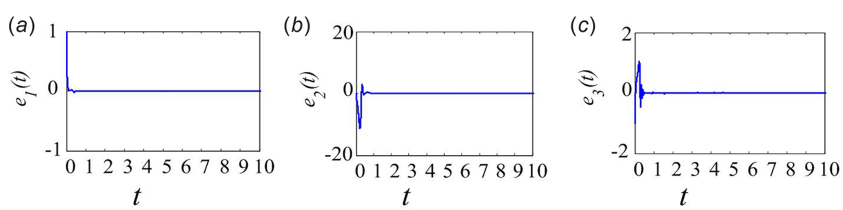

Fig. 16 **The simulations of the errors between the drive system (22) and the response system (23) when C=diag$\{-25, -25, -25\}$ and $\alpha = 2$: (*a*) the evolution of e_1, (*b*) the evolution of e_2, and (*c*) the evolution of e_3**

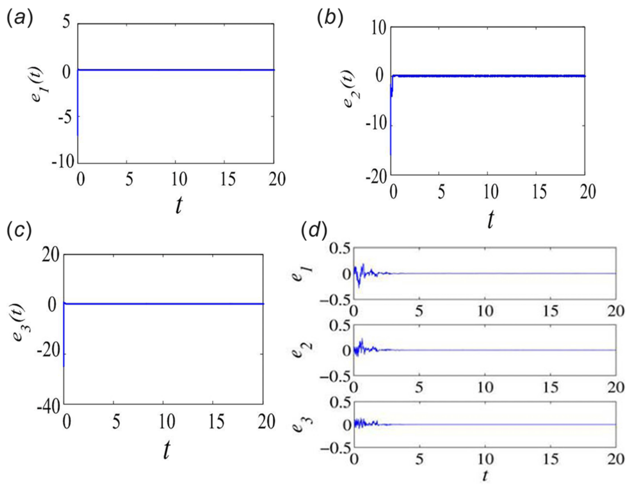

Fig. 18 The simulations of the errors between the drive system (22) and the response system (23) when $C = \mathrm{diag}\{-200, -200, -200\}$ and $\alpha = 10$: (*a*) the evolution of e_1, (*b*) the evolution of e_2, (*c*) the evolution of e_3, and (*d*) the simulations of the errors [29]

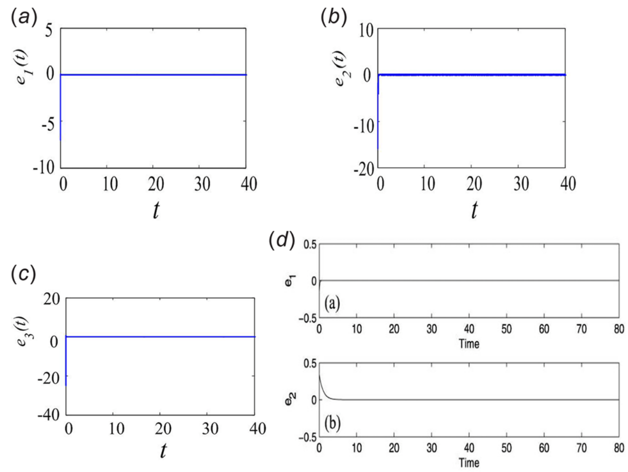

Fig. 20 The simulations of the errors between the drive system (22) and the response system (23) when $C = \mathrm{diag}\{-200, -200, -200\}$ and $\alpha = 10$: (*a*) the evolution of e_1, (*b*) the evolution of e_2, (*c*) the evolution of e_3, and (*d*) the simulations of the errors [31]

by setting different constant matrix *C*. Last but not least, comparing with other papers mentioned above in our paper, by enlarging the absolute value of product of the diagonal elements of diagonal matrix *C*, we can shorten the time of synchronization in this paper. Meanwhile, the curves of chaotic synchronization are smoother than the ones in other synchronization methods.

4.2 Comparative Results With Other Methods. In the section, first, we will verify that the larger the absolute value of product of the diagonal elements of diagonal matrix is, the smoother the curve of chaotic synchronization is and the shorter the time of chaotic synchronization is in this paper. Second, we will compare the time of chaotic synchronization with adaptive synchronization [28], complete synchronization [29], coupled synchronization [30], function projective synchronization [31], and phase synchronization [32]. Table 1 is drawn to show the approximate time of errors systems tending to zero compared with other papers.

Figures 14–16 show the larger the absolute value of product of the diagonal elements of diagonal matrix is, the smoother the curve of chaotic synchronization is and the shorter the time of synchronization is in this paper. Figures 17–21 show that the synchronization time of chaotic system is shorter, and the curve of chaotic synchronization in this paper is smoother than the ones in other methods.

5 Conclusion

In this paper, we have proposed the active adaptive projective synchronization scheme for the identical chaotic synchronization and applied it to adaptive projective-synchronize two identical chaotic systems. In our active controlling scheme, we expand the controlling capability by using a constant diagonal matrix, which can be adjustable. So, we can control the synchronization more flexibly. Based on our synchronization, we have designed a feedback controller and the parameters update laws. Feasibility of the technique is illustrated for the Lorenz system. It implies that the two identical chaotic systems achieve adaptive projective

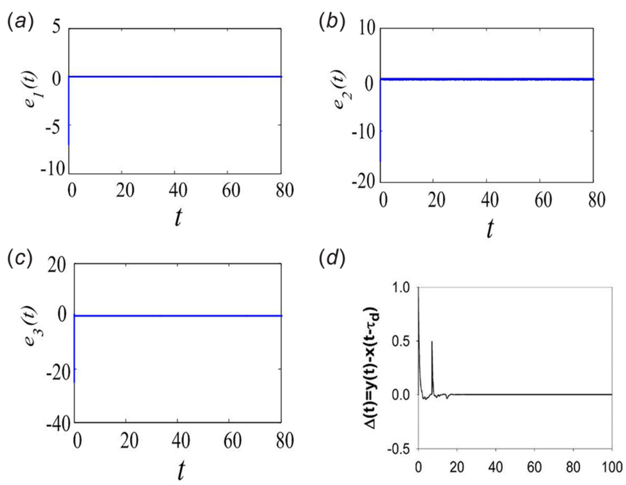

Fig. 19 The simulations of the errors between the drive system (22) and the response system (23) when $C = \mathrm{diag}\{-200, -200, -200\}$ and $\alpha = 10$: (*a*) the evolution of e_1, (*b*) the evolution of e_2, (*c*) the evolution of e_3, and (*d*) The simulations of the errors [30]

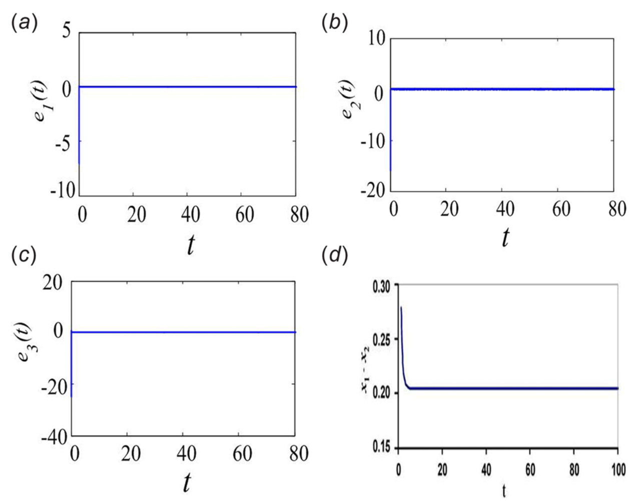

Fig. 21 The simulations of the errors between the drive system (22) and the response system (23) when $C = \mathrm{diag}\{-200, -200, -200\}$ and $\alpha = 10$: (*a*) the evolution of e_1, (*b*) the evolution of e_2, (*c*) the evolution of e_3, and (*d*) the simulations of the errors [32]

synchronization based on active control. More importantly, in the above simulations, the synchronization between the drive system and the response system can be achieved by taking different constant matrix C. Therefore, the tunability of the chaotic synchronization is verified. Moreover, in Sec. 4.2, comparative results show that we can shorten the time of chaotic synchronization and make the curve of chaotic synchronization smoother than the ones in other synchronization methods by enlarging the absolute value of product of the diagonal elements of diagonal matrix $\mathbf{C}$.

Furthermore, with the help of feedback control and the parameters update laws, the drive system (22) and the response system (23) realize active adaptive projective synchronization and the response system (23) adapt themselves to the true values. Numerical MATLAB simulations verify the effectiveness of the proposed synchronization scheme.

Funding Data

- National Natural Science Foundation of China (Grant No. 61571185).
- The Natural Science Foundation of Hunan Province, China (Grant No. 2016JJ2030).
- Open Fund Project of Key Laboratory in Hunan Universities (Grant No. 15K027).

References

[1] Pecora, L. M., and Carroll, T. L., 1990, "Synchronization of Chaotic Systems," Phys. Rev. Lett., **64**(8), pp. 821–830.
[2] Akbarzadeh-T, M.-R., Hosseini, S. A., and Naghibi-Sistani, M.-B., 2017, "Stable Indirect Adaptive Interval Type-2 Fuzzy Sliding-Based Control and Synchronization of Two Different Chaotic Systems," Appl. Soft Comput., **55**(1), pp. 576–587.
[3] Hosseini, S. A., Akbarzadeh-T, M.-R., and Naghibi-Sistani, M.-B., 2013, "A Synchronizing Controller Using a Direct Adaptive Interval Type-2 Fuzzy Sliding Mode Strategy," IEEE International Conference on Fuzzy Systems (FUZZ), Hyderabad, India, July 7–10.
[4] Junan, L., Xiaoqun, W., and Lü, J., 2002, "Synchronization of a Unified Chaotic System and the Application in Secure Communication," Phys. Lett. A, **305**(6), pp. 365–370.
[5] Bowong, S., 2004, "Stability Analysis for the Synchronization of Chaotic Systems With Different Order: Application to Secure Communications," Phys. Lett. A, **326**(1–2), pp. 102–113.
[6] Nan, M., Wong, C.-N., Tsang, K.-F., and Shi, X., 2000, "Secure Digital Communication Based on Linearly Synchronized Chaotic Maps," Phys. Lett. A, **268**(1–2), pp. 61–68.
[7] Osipov, G. V., Pikovsky, A. S., and Kurths, J., 2002, "Phase Synchronization of Chaotic Rotators," Phys. Rev. Lett., **88**(5), p. 054102.
[8] Xu, Y., Zhou, W., Fang, J., and Sun, W., 2010, "Adaptive Lag Synchronization and Parameters Adaptive Lag Identification of Chaotic Systems," Phys. Lett. A, **374**(34), pp. 3441–3446.
[9] Liping, C., Yi, C., and Ranchao, W., 2011, "Lag Projective Synchronization in Fractional-Order Chaotic (Hyperchaotic) Systems," Phys. Lett. A, **375**(21), pp. 2099–2110.
[10] Chenggui, Y., Qi, Z., and Jun, Y., 2013, "Complete Synchronization Induced by Disorder in Coupled Chaotic Lattices," Phys. Lett. A, **377**(5), pp. 370–377.
[11] Jiang, G.-P., Tang, W. K.-S., and Chen, G., 2003, "A Simple Global Synchronization Criterion for Coupled Chaotic Systems," Chaos, Solitons Fractals, **15**(5), pp. 925–935.
[12] Nobukawa, S., Nishimura, H., Yamanishi, T., and Liu, J.-Q., 2015, "Analysis of Chaotic Resonance in Izhikevich Neuron Model," PloS One, **10**(9), p. e0138919.
[13] Nobukawa, S., Nishimura, H., and Yamanishi, T., 2017, "Chaotic Resonance in Typical Routes to Chaos in the Izhikevich Neuron Model," Sci. Rep., **7**(1), p. 1331.
[14] Meng, J., and Wang, X., 2007, "Nonlinear Observer Based Phase Synchronization of Chaotic Systems," Phys. Lett. A, **369**(4), pp. 294–298.
[15] Ghosh, D., 2010, "Projective Synchronization in Multiple Modulated Time-Delayed Systems With Adaptive Scaling Factor," Nonlinear Dyn., **62**(4), pp. 751–759.
[16] Pourdehi, S., Karimaghaee, P., and Karimipou, D., 2011, "Adaptive Controller Design for Lag-Synchronization of Two Non-Identical Time-Delayed Chaotic Systems With Unknown Parameters," Phys. Lett. A, **375**(17), pp. 1769–1778.
[17] Sang, J., Yang, J., and Yue, L., 2011, "Complete Synchronization of Double-Delayed Rössler Systems With Uncertain Parameters," Chin. Phys. B, **20**(8), p. 080507.
[18] Bhowmick, S. K., Hens, C., Ghosh, D., and Dana, S. K., 2012, "Mixed Synchronization in Chaotic Oscillators Using Scalar Coupling," Phys. Lett. A, **376**(36), pp. 2490–2495.
[19] Wang, CHua., Yan, H., Fei, Y., and Hao, X., 2013, "Time-Controllable Projective Synchronization of a Class of Chaotic Systems Based on Adaptive Method," Acta Phys. Sin., **62**(11), pp. 139–145.
[20] Li, N., and Li, J.-F., 2011, "Unified Projective Synchronization of Chaotic System," Acta Phys. Sin., **60**(11), p. 110512.
[21] Li, Z.-B., Zhao, X.-S., and Wang, J., 2011, "Generalized Projective Synchronization of Chaotic Systems Via Modified Active Control," Acta Phys. Sin., **60**(5), pp. 107–114.
[22] Wang, X., Nian, F., and Guo, G., 2009, "High Precision Fast Projective Synchronization in Chaotic (Hyperchaotic) Systems," Phys. Lett. A, **373**(20), pp. 1754–1761.
[23] Chunmei, L., 2010, "Application of Lyapunov Approach on Stability Theory of System," Master's thesis, Northeast Normal University, Changchun, China.
[24] Lorenz, E. N., 1963, "Deterministic Nonperiodic Flow," J. Atmos. Sci., **20**(20), pp. 130–141.
[25] Samaneh, M., and Tahereh, B., 2018, "Robust Adaptive Synchronization of Chaotic Systems With Nonsymmetric Input Saturation Constraints," ASME J. Comput. Nonlinear Dynam., **13**(1), p. 011005.
[26] Liu, H.-J., Yu, H., and Zhu, Z.-L., 2017, "A Special Hybrid Projective Synchronization in Symmetric Chaotic System With Unknown Parameter," ASME J. Comput. Nonlinear Dynam., **12**(5), p. 051015.
[27] Tang, R.-A., Liu, Y.-L., and Xue, J.-K., 2009, "An Extended Active Control for Chaos Synchronization," Phys. Lett. A, **373**(16), pp. 1449–1454.
[28] Zhang, H., Huang, W., Wang, Z., and Chai, T., 2006, "Adaptive Synchronization Between Two Different Chaotic Systems With Unknown Parameters," Phys. Lett. A, **350**(5–6), pp. 363–366.
[29] Zhou, J., and Chen, Z., 2008, "Further Results on Complete Synchronization for Noise-Perturbed Chaotic Systems," Phys. Lett. A, **372**(33), pp. 5394–5401.
[30] Hoang, T. M., and Nakagawa, M., 2006, "Synchronization of Coupled Nonidentical Multidelay Feedback Systems," Phys. Lett. A, **363**(3), pp. 218–224.
[31] Du, H., Zeng, Q., and Wang, C., 2008, "Function Projective Synchronization of Different Chaotic Systems With Uncertain Parameters," Phys. Lett. A, **372**(33), pp. 5402–5410.
[32] Erjaee, G. H., and Momani, S., 2008, "Phase Synchronization in Fractional Differential Chaotic Systems," Phys. Lett. A, **372**(14), pp. 2350–2354.